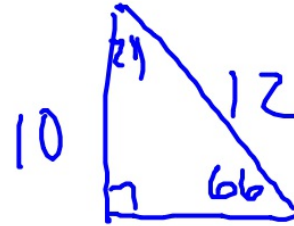


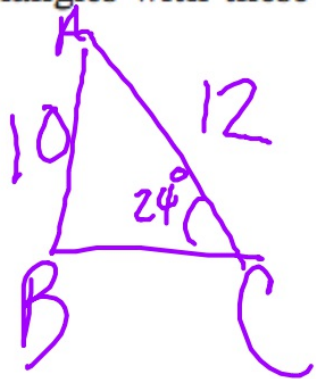
IB Math Studies 2 Bell Work

$$\# \left[ \frac{10}{\sin 66} = \frac{\sqrt{12^2 - 10^2}}{\sin 24} \right]$$



$$\sin 66 = \frac{10}{12}$$

A triangle ABC has  $AB = 10$  cm,  $AC = 12$  cm and  $\widehat{ACB} = 24^\circ$ . It is possible to draw two different triangles with these measurements. Calculate the two possible values for  $\widehat{ABC}$ .



$$\frac{\sin 24}{10} = \frac{\sin x}{12}$$

$$x = 29.2^\circ \text{ \& } 151^\circ$$

$$24^\circ, 29.2^\circ, \text{---}^\circ$$

$$24^\circ, 151^\circ, \text{---}^\circ$$

**Chapter**

**11**

## **Two variable statistics**

- C** Line of best fit by eye
- D** Linear regression

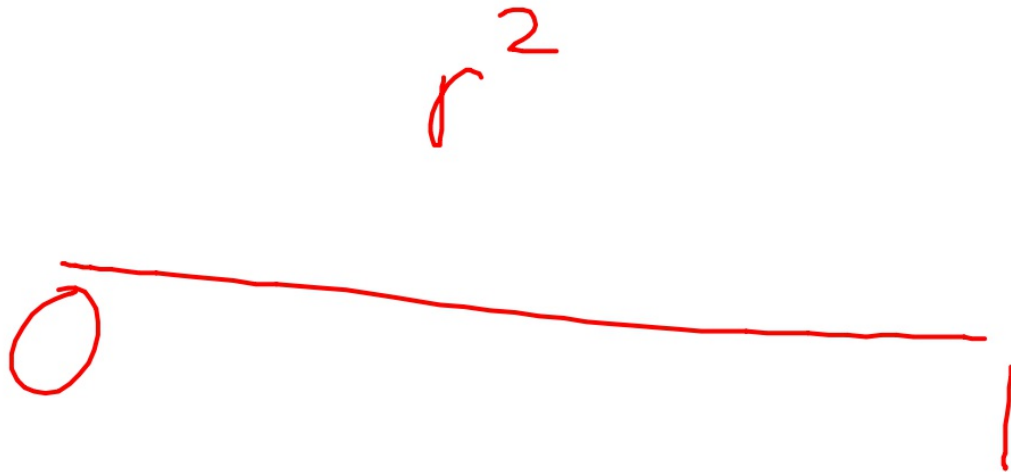
**Syllabus reference: 4.2, 4.3, 4.4**

## THE COEFFICIENT OF DETERMINATION $r^2$ (EXTENSION)

To help describe the correlation between two variables, we can also calculate the **coefficient of determination**  $r^2$ . This is simply the square of Pearson's correlation coefficient  $r$ , and as such the direction of correlation is eliminated.

Given a set of bivariate data, we can find  $r^2$  using our calculator in the same way we find  $r$ .

Alternatively, if  $r$  is already known, we can simply square this value.



## INTERPRETATION OF THE COEFFICIENT OF DETERMINATION

If there is a causal relationship then  $r^2$  indicates the degree to which change in the independent variable explains change in the dependent variable.

For example, an investigation into many different brands of muesli found that there is strong positive correlation between the variables *fat content* and *kilojoule content*. It was found that  $r \approx 0.862$  and  $r^2 \approx 0.743$ .

An interpretation of this  $r^2$  value is:

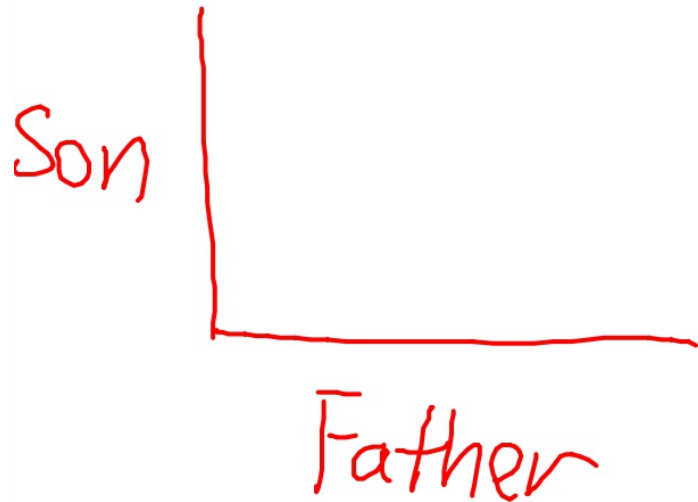
74.3% of the variation in *kilojoule content* of muesli can be explained by the variation in *fat content* of muesli.

The diagram consists of two red arrows pointing from labels above to terms in the text. One arrow points from 'dependent variable' to 'kilojoule content'. Another arrow points from 'independent variable' to 'fat content'. There is also a red line under the word 'muesli' in the first sentence of the paragraph.

At a father-son camp, the heights of the fathers and their sons were measured.

Father's height ( $x$ cm)	175	183	170	167	179	180	183	185	170	181	185
Son's height ( $y$ cm)	167	178	158	162	171	167	180	177	152	164	172

- ~~a Draw a scatter diagram of the data.~~
- b Calculate  $r^2$  for the data and interpret its value.



$$r = 0.8265$$

$$r^2 = 0.683$$

68.3% of the variation in sons' heights can be explained by the variation in fathers' heights.

# C

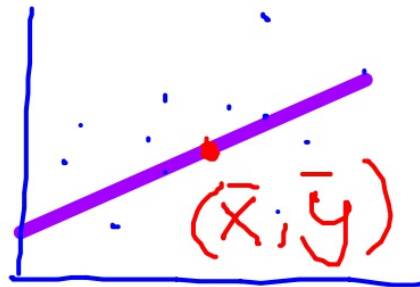
## LINE OF BEST FIT BY EYE

If two variables have a strong linear relationship, we can draw a best fit line to illustrate their relationship. The line drawn is called a line of best fit by eye and varies from person to person.

*Step 1:* Calculate the mean of the  $X$  values  $\bar{x}$ , and the mean of the  $Y$  values  $\bar{y}$ .

*Step 2:* Mark the mean point  $(\bar{x}, \bar{y})$  on the scatter diagram.

*Step 3:* Draw a line through the mean point which fits the trend of the data, and so that about the same number of data points are above the line as below it.



On a hot day, six cars were left in the sun in a car park. The length of time each car was left in the sun was recorded, as well as the temperature inside the car at the end of the period.



<i>Car</i>	A	B	C	D	E	F
<i>Time <math>x</math> (min)</i>	50	5	25	40	15	45
<i>Temperature <math>y</math> (<math>^{\circ}\text{C}</math>)</i>	47	28	36	42	34	41

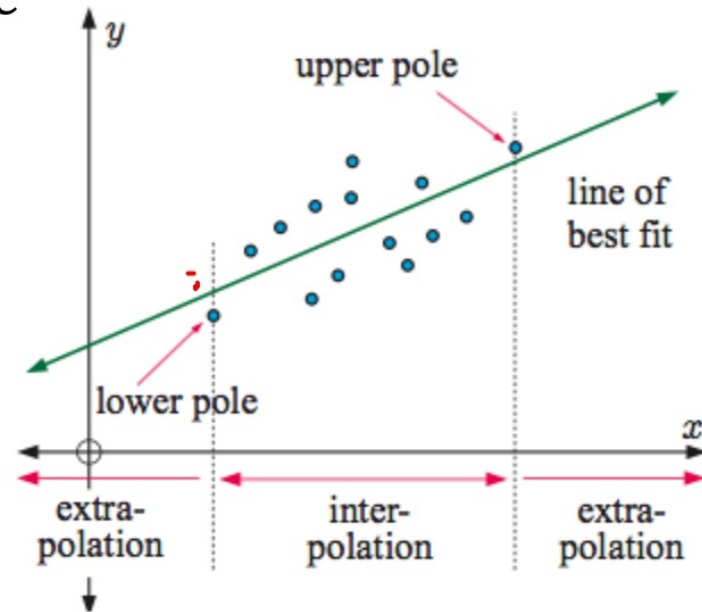
- a** Calculate  $\bar{x}$  and  $\bar{y}$ .
- b** Draw a scatter diagram for the data.
- c** Plot the mean point  $(\bar{x}, \bar{y})$  on the scatter diagram. Draw a line of best fit through this point.
- d** Predict the temperature of a car which has been left in the sun for:
  - i** 35 minutes
  - ii** 75 minutes.
- e** Comment on the reliability of your predictions in **d**.

# INTERPOLATION AND EXTRAPOLATION

Consider the data in the scatter diagram alongside. The data with the highest and lowest values are called the poles.

If we predict a value using the data between the poles, we are *interpolating*.

If we predict a value using the data trend outside the poles, we are *extrapolating*.





# INTERPOLATION AND EXTRAPOLATION

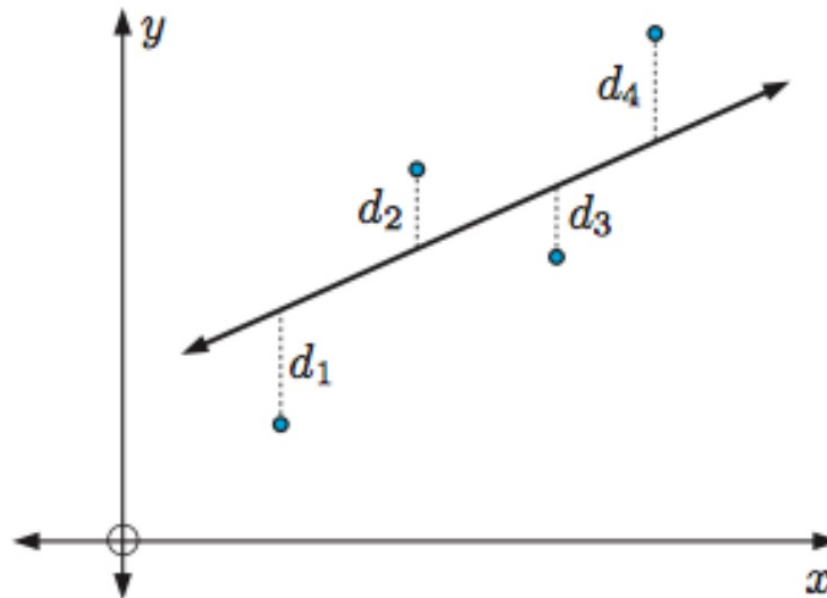
The accuracy of an interpolation depends on how linear the original data was. This can be gauged by the correlation coefficient and by being sure the line of fit is accurate to the data.

The accuracy of extrapolation depends not only on how linear the original data was, but also on the assumption that the linear trend will continue past the poles. The validity of this assumption depends greatly on the situation.

# D

## LINEAR REGRESSION

Lines of best fit by eye can vary. To calculate the line that most accurately fits the data, we run a linear regression. The most common linear regression is the Least Squares Regression. This is done in the calculator.



Homework:

Exercises 11 C #1  
11 D #1, 6