A man holds two tickets in a 100-ticket lottery in which there are two winning tickets. If no replacement occurs, determine the probability that he will win:

a both prizes

$\frac{2}{100} \times \frac{1}{99}$

$\frac{1}{4950}$

b neither prize

$\frac{98}{100} \times \frac{97}{99}$

$\frac{4753}{4950} = 0.96$

c at least one prize.

$1 - P(\text{does not win either})$

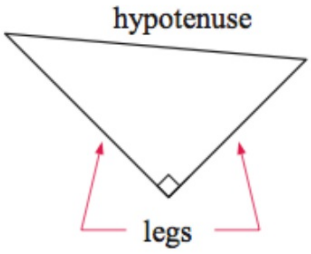
$1 - 0.96$

Chapter 12

Pythagoras' theorem

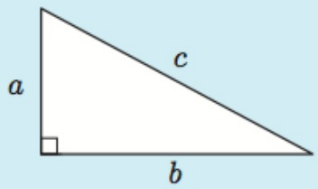
Contents:

A PYTHAGORAS' THEOREM



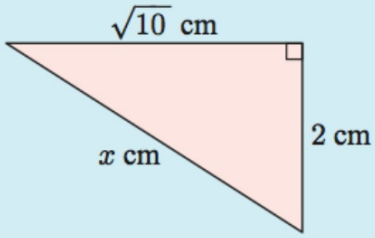
In a right angled triangle with legs a and b, and hypotenuse c,

$$a^2 + b^2 = c^2$$



Example 3

Find x in the following:



$$x^2 = (\sqrt{10})^2 + 2^2$$

$$x^2 = 10 + 4$$

$$\sqrt{x^2} = \sqrt{14}$$

$$x = \pm\sqrt{14}$$

$$x = \sqrt{14} \approx 3.74$$

ACCURACY OF ANSWERS

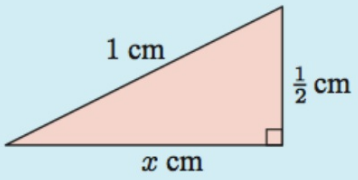
In **Example 1**, the solution $\sqrt{13}$ in surd form is exact, and is acceptable since it is irrational. If the answer was $\sqrt{16}$, you would be expected to simplify it to 4.

Answers given in surd form may not always be practical in real contexts. For example, if we needed to draw a line $\sqrt{13}$ centimetres long using a ruler, we would approximate the value to 3.6 cm using a calculator.

Within all IB Mathematics courses, final answers should be given either exactly or correct to 3 significant figures. Rounding to 3 significant figures should only occur at the end of a calculation and not at intermediate steps.

Example 4

Solve for x:



$$\left(\frac{1}{2}\right)^2 + x^2 = 1^2$$

$$\frac{1}{4} + x^2 = 1$$

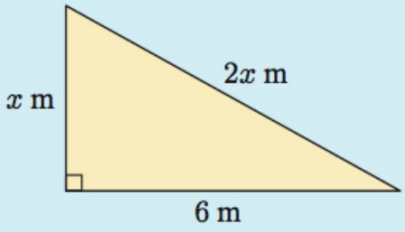
$$x^2 = \frac{3}{4}$$

$$x = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$\approx 0.866$$

Example 5

Find the value of x:



$$x^2 + 6^2 = (2x)^2$$

$$x^2 + 36 = 4x^2$$

$$-x^2 \qquad -x^2$$

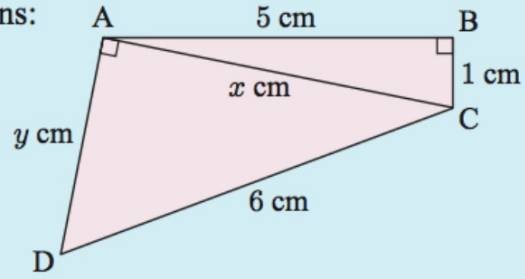
$$\frac{36}{3} = \frac{3x^2}{3}$$

$$12 = x^2$$

$$x = \sqrt{12} \approx 3.46$$

Example 6 Self Tutor

Find the value of any unknowns:



$x = \sqrt{26}$ cm

$y = \sqrt{10}$ cm

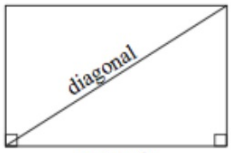
Assignment:

Exercise 12 A

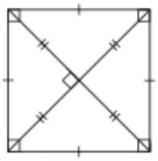
3 a-c,
4 e-f
5 d-f
6,7,8,9 all

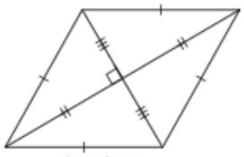
B RIGHT ANGLES IN GEOMETRY

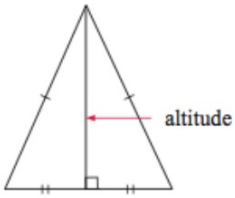
In many geometric figures, we can make right-angled triangles and thus use the Pythagorean Theorem to find lengths.



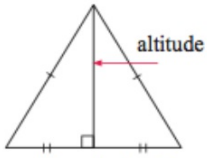
rectangle


square


rhombus

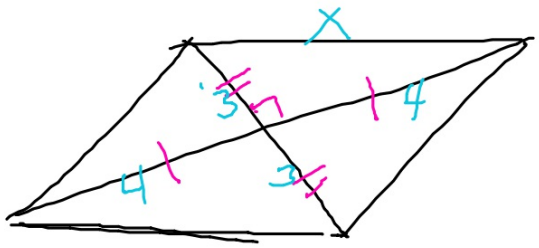


isosceles triangle



equilateral triangle

A rhombus has diagonals of length 6 cm and 8 cm. Find the length of its sides.



5 cm

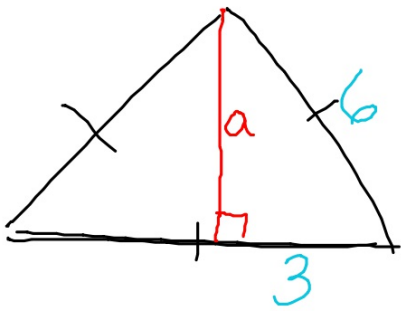
$3^2 + 4^2 = x^2$

$9 + 16 = x^2$

$25 = x^2$

a Find the altitude of an equilateral triangle with sides 6 m long.

b Hence find the area of the triangle.



$a = \sqrt{27}$ m

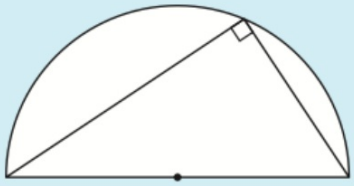
$A = \frac{1}{2}bh$

$\frac{1}{2}(6)(\sqrt{27})$

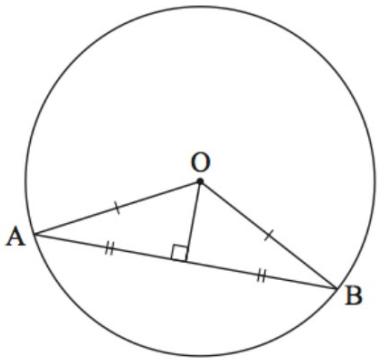
$A = 7.79\ m^2$

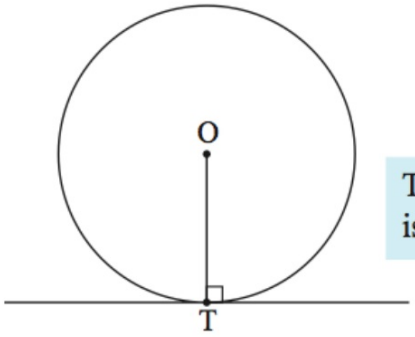
Right angles in circles:

The angle in a semi-circle is always a right angle.



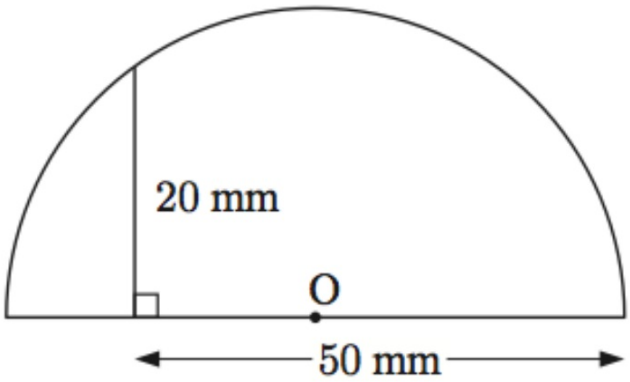
The line drawn from the centre of a circle at right angles to a chord, bisects the chord.





The radius from the centre of a circle to the point of contact is at right angles to the tangent.

Find the radius of this semi-circle.

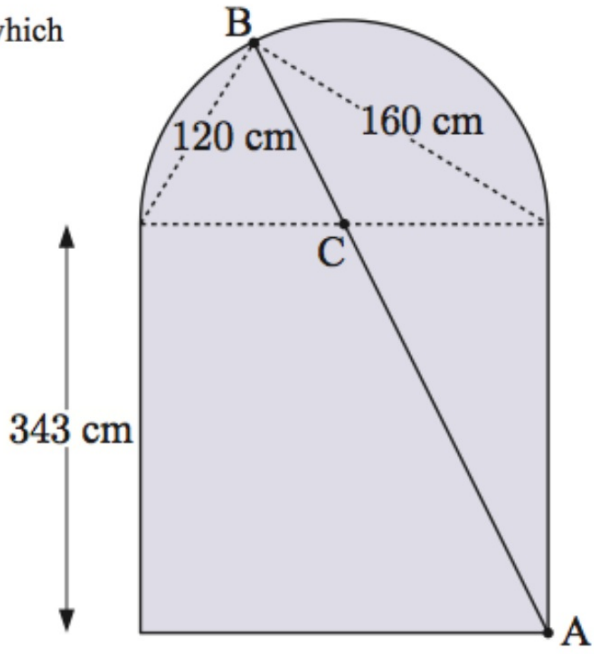


The doorway alongside is rectangular at the bottom with a semi-circular arch at the top.

a Find the radius of the semi-circle.

b Find the length of line segment AC.

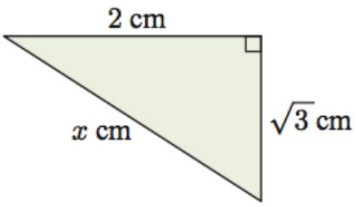
c Hence find the length of the line segment AB which passes through the circle's centre C.



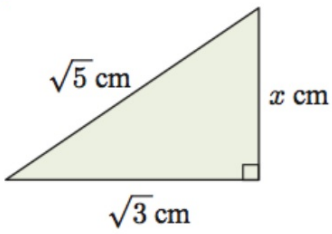
Exercise 12 A # 3 a-c, # 4 e-f, # 5 d-f, # 6,7,8,9 all

3 Find x in the following:

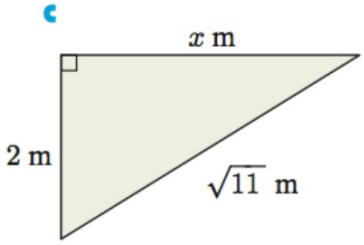
a



b

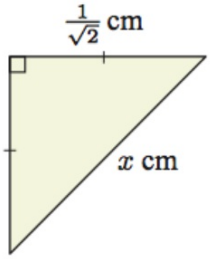


c

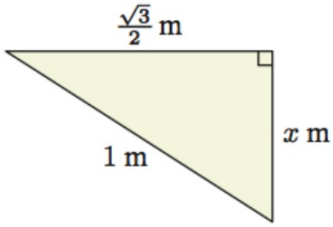


4 Solve for x:

e

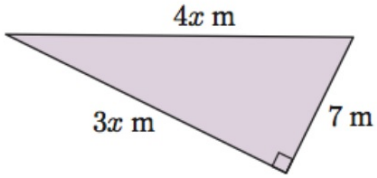


f

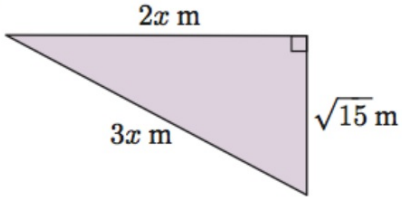


5 Find the value of x:

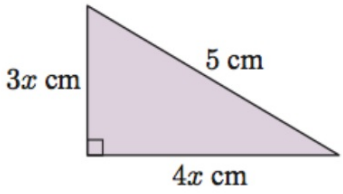
d



e

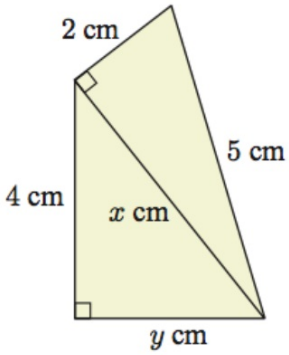


f

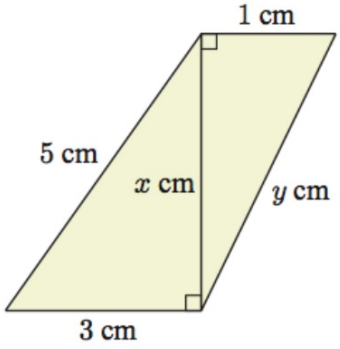


6 Find the value of any unknowns:

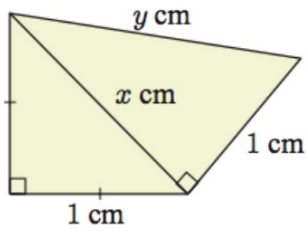
a



b

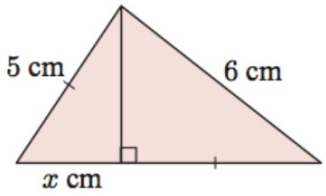


c

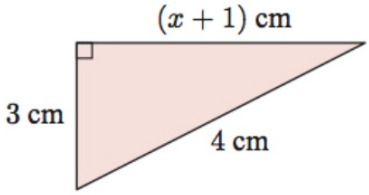


7 Find x:

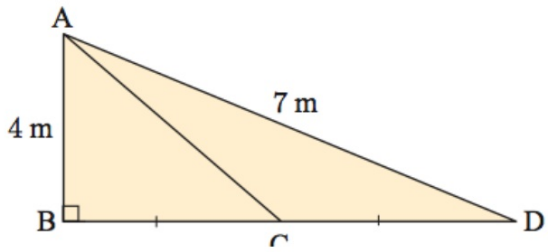
a



b

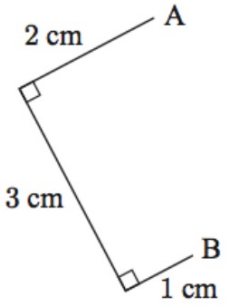


8 Find the length AC:



9 In the following figures, draw additional lines to complete right angled triangles. Apply Pythagoras' theorem to find the unknown distance AB.

a



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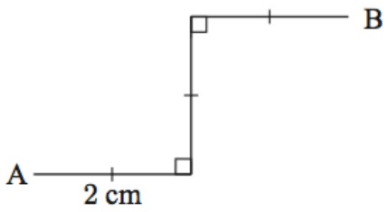

c