OPENING PROBLEM

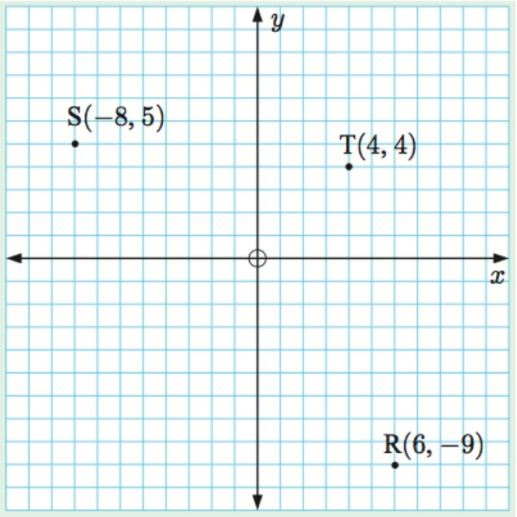
A city has two hospitals: Ridgehaven located at R(6, −9), and Sunport located at S(−8, 5).

Things to think about:

a Trish lives at T(4, 4). Which hospital is Trish closest to?

b Can you find the point midway between the hospitals?

c The city's planning council wants to define a 'boundary line' so that people will go to the hospital closest to them. Can you find the equation of this boundary line?



Circles:

The distance between a point and a line is the length of the segment perpendicular to the line.

If a radius/diameter is ⊥ to a chord, it bisects the chord

Chapter 13

Coordinate geometry

Syllabus reference: 5.1

Contents:

A DISTANCE BETWEEN TWO POINTS

The distance d between two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

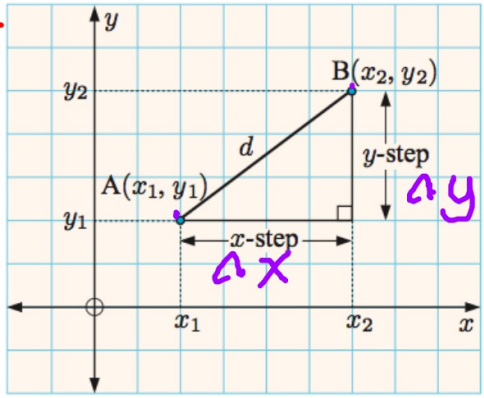
$\Delta x = (x_2 - x_1)$

$\Delta y = (y_2 - y_1)$

Let's derive the distance formula:

$\sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{d^2}$

$\sqrt{(\Delta x)^2 + (\Delta y)^2} = d$



Use the distance formula to classify triangle ABC as either equilateral, isosceles, or scalene.

$x_1 y_1$ $x_2 y_2$

3 same 2 same 3 different

a A(−1, 0), B(−2, 3), C(−5, 4)

$$AB = \sqrt{(-2--1)^2+(3-0)^2}$$

$$\sqrt{1+9} = \sqrt{10} = AB$$

BC

AC

Use the distance formula to see if the following triangles are right angled. If they are, state the vertex where the right angle is.

b A(−1, 2), B(3, 4), C(5, 0)

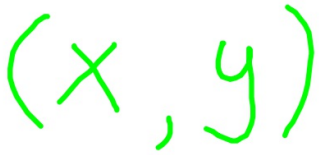
find all 3 side lengths

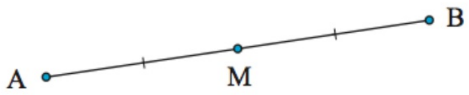
if $a^2+b^2=c^2$ (longest side), then

it is a right $\triangle$

B MIDPOINTS

The point M halfway between points A and B is called the **midpoint** of line segment AB.





M is the midpoint of AB.

The x-coordinate of M is the *average* of the x-coordinates of A and B.

The y-coordinate of M is the *average* of the y-coordinates of A and B.

THE MIDPOINT FORMULA

The coordinates of the midpoint of the line segment with endpoints (x_1, y_1) and (x_2, y_2) are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

M is the midpoint of AB. Find the coordinates of B if:

e A is $(0, 0)$ and M is $(2, -\frac{1}{2})$

$2+2=4$

$-\frac{1}{2}+-\frac{1}{2}=-1$

$(4,-1)$

f A is $(-3, \frac{1}{2})$ and M is $(0, 0)$

$2=\frac{x_1+0}{2}$

$4=x_1$

$-\frac{1}{2}=\frac{y_1+0}{2}$

$-1=y_1$ $(4,-1)$

AB is a diameter of a circle, centre $(3\frac{1}{2}, -1)$. Given that B is $(2, 0)$, find the coordinates of A.

M $x_2\ y_2$ (x_1, y_1)