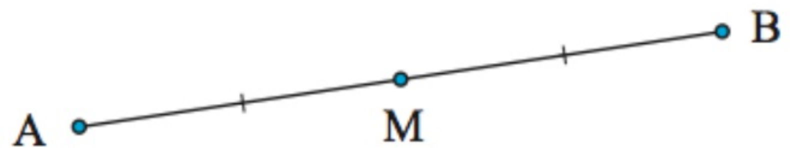
# B MIDPOINTS

The point M halfway between points A and B is called the **midpoint** of line segment AB.




M is the midpoint of AB.

The $x$-coordinate of M is the *average* of the $x$-coordinates of A and B.

The $y$-coordinate of M is the *average* of the $y$-coordinates of A and B.

## THE MIDPOINT FORMULA

The coordinates of the midpoint of the line segment with endpoints $(x_1, y_1)$ and $(x_2, y_2)$ are

$$\left(\frac{\boldsymbol{x_1 + x_2}}{\mathbf{2}}, \frac{\boldsymbol{y_1 + y_2}}{\mathbf{2}}\right).$$

M is the midpoint of AB. Find the coordinates of B if:

A is $(-2, 1)$ and M is $(-1\frac{1}{2}, 3)$

$$\cancel{2}\left(\frac{-2+a}{\cancel{2}}\right)=\left(-1\tfrac{1}{2}\right)2$$

$$-2+a=-3$$

$$+2 \qquad +2$$

$$a=-1$$

$$\frac{1+b}{2}=3$$

A is $(1, 3)$ and M is $(2, -1)$

$$\left(\frac{1+x}{2}\right)=2$$

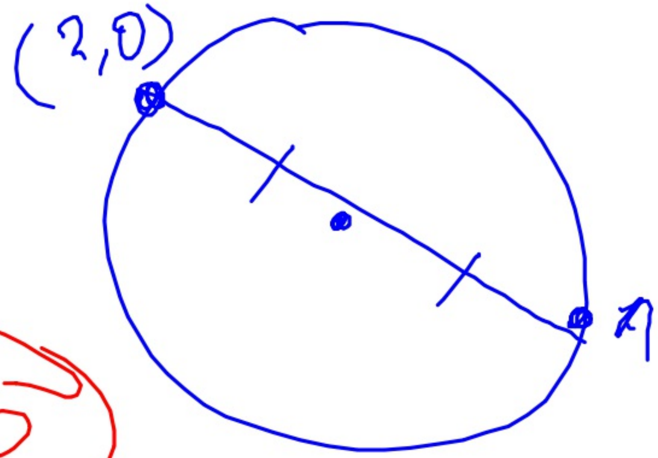
$$x=3$$

$$\left(\frac{3+y}{2}\right)=-1$$

$$y=-5$$

AB is a diameter of a circle, centre $(3\frac{1}{2}, -1)$. Given that B is $(2, 0)$, find the coordinates of A.

$A\ (x, y)$

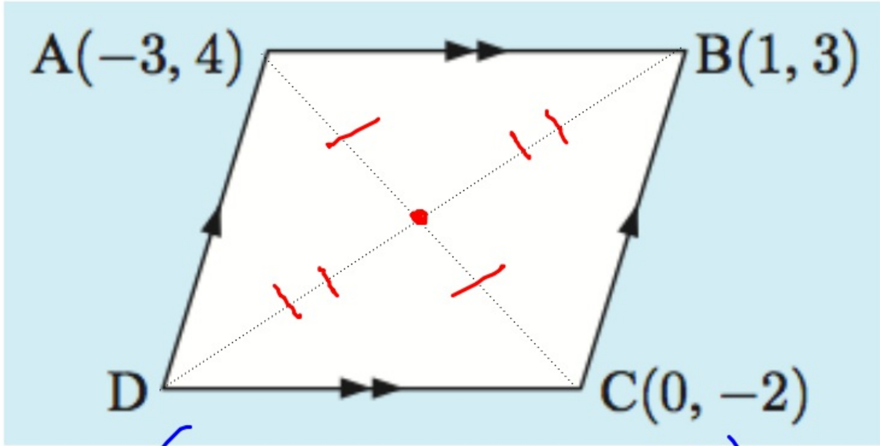



$$2\left(\frac{x+2}{2}\right) = \left(3\frac{1}{2}\right)2$$

$$x+2 = 7 \quad x = 5$$

$$\frac{y+0}{2} = -1$$

Use midpoints to find the fourth vertex of the given parallelogram:




Find the midpoint of $\overline{AC}$. This is also the midpoint of $\overline{BD}$. Use it to find the coordinates of D.

$$\left(\frac{-3+0}{2}, \frac{4+(-2)}{2}\right)$$

$$\left(-1\tfrac{1}{2}, 1\right)$$

$D(x, y)$

$$\frac{1+x}{2} = -1\tfrac{1}{2} \qquad \frac{3+y}{2} = 1$$

$$x = -4 \qquad y = -1$$

Assignment:

Exercise 13 A # 5a-b, #6d-e
13 B # 3e-f, # 6, #10, #12b-c

## C GRADIENT

The **gradient** of a line is a measure of its steepness.

The **gradient** of a line $= \dfrac{\text{vertical step}}{\text{horizontal step}}$ or $\dfrac{\boldsymbol{y}\textbf{-step}}{\boldsymbol{x}\textbf{-step}}$.

- the gradient of **horizontal** lines is **0**
- the gradient of **vertical** lines is **undefined**.

The **gradient** $m$ of the line passing through $(x_1, y_1)$ and $(x_2, y_2)$ is $\boldsymbol{m = \dfrac{y_2 - y_1}{x_2 - x_1}}$.

Find $a$ given that the line joining:

A($a$, 8) to B(−3, −4) has gradient $\frac{2}{3}$.

$x_1 \ y_1 \qquad x_2 \ y_2$

$$\frac{-12}{} \quad \frac{-4-8}{-3-a} = \frac{2}{3}$$

$$\frac{-12}{-3-a} = \frac{2}{3}$$

$$-36 = 2(-3-a)$$

$$-36 = -6-2a$$

$$+6 \qquad +6$$

$$-30 = -2a$$

$$a = 15$$

## D PARALLEL AND PERPENDICULAR LINES

### PARALLEL LINES

have the same slope

For lines $l_1$ and $l_2$ with gradients $m_1$ and $m_2$ respectively, $l_1$ is parallel to $l_2 \Leftrightarrow m_1 = m_2$.

## PERPENDICULAR LINES

have negative reciprocal slope

For non-vertical and non-horizontal lines $l_1$ and $l_2$ with gradients $m_1$ and $m_2$ respectively,

$$l_1 \text{ is perpendicular to } l_2 \Leftrightarrow m_1 \times m_2 = -1.$$

Consider the points A(1, 2), B(−3, 0), C(5, 3), and D(3, $k$). Find $k$ if:

**a** AB is parallel to CD

$$\frac{0-2}{-3-1} = \frac{-2}{-4} = \frac{1}{2}$$

$$\frac{k-3}{3-5} = \frac{1}{2}$$

$$\frac{-1}{-2} = \frac{k-3}{-2} = \frac{1}{2}$$

$$k = 2$$

**c** AB is perpendicular to CD

$$m = -2$$

$$-2\left(\frac{k-3}{3-5}\right) = (-2)\,{-2}$$

$$k-3 = 4$$

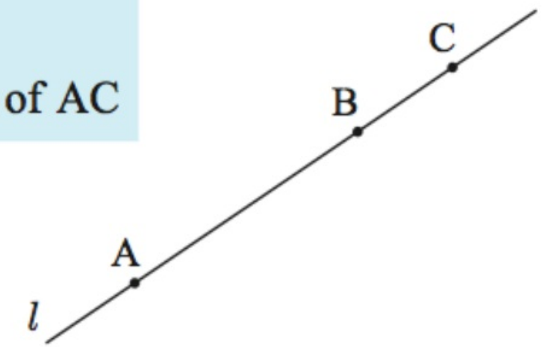
$$k = 7$$

## COLLINEAR POINTS

Three or more points are **collinear** if they lie on the same straight line.

Three points A, B, and C are **collinear** if:

$$\text{gradient of AB} = \text{gradient of BC} = \text{gradient of AC}$$



Determine whether the following sets of three points are collinear:

A(–1, 7), B(1, 1), C(4, –8)

slope of AB = slope of BC = slope of AC

$$\frac{1-7}{1-(-1)} = \frac{-6}{2} \qquad \frac{-8-1}{4-1} = \frac{-9}{3} \qquad \frac{-8-7}{4-(-1)} \quad \frac{-15}{5}$$

$$-3 \qquad -3 \qquad -3$$

Yes

Assignment:

Exercise ~~13 A # 5a-b, #6d-e~~
13 B # 3e-f, # 6, #10, #12b-c
13 C.2 # 3b-c,