Chapter 17

Quadratic functions

Syllabus reference: 1.6, 6.3

You should be able to identify the following features from a graph and an equation.

line of symmetry
vertex
x-intercept(s)
y-intercept

You will also need to be able to write the equation given this information.

D GRAPHS OF THE FORM $y = ax^2$

If $a > 0$, $y = ax^2$ opens upwards. It has the shape 

If $a < 0$, $y = ax^2$ opens downwards. It has the shape 

If $a < -1$ or $a > 1$, $y = ax^2$ is 'thinner' than $y = x^2$.

If $-1 < a < 1$, $a \neq 0$, $y = ax^2$ is 'wider' than $y = x^2$.

The vertex of $y = ax^2$ is $(0, 0)$. The graph is always symmetrical.

E GRAPHS OF QUADRATIC FUNCTIONS

Sketch the graph of $y = x^2 - 2x - 3$ by considering:

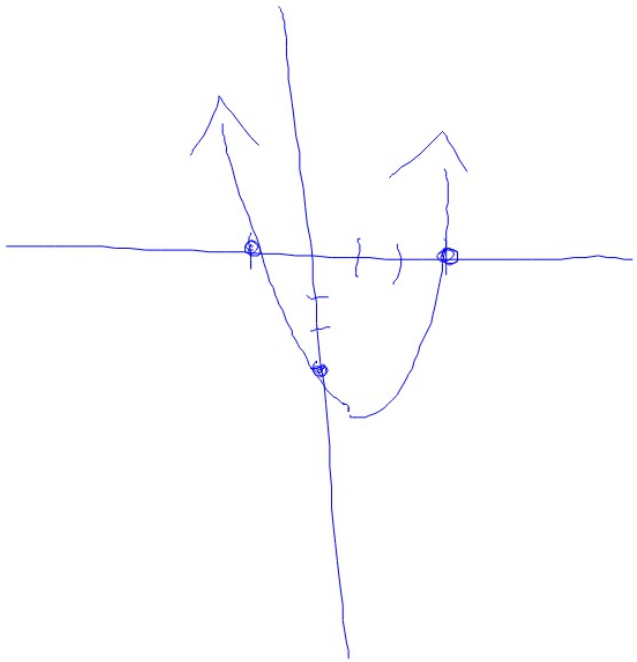
a the value of a **b** the y-intercept **c** the x-intercepts.

$a = 1$

-3

$(x-3)(x+1)$

$3, -1$



Sketch the graph of $y = -2(x+1)(x-2)$ by considering:

a the value of a **b** the y-intercept **c** the x-intercepts.

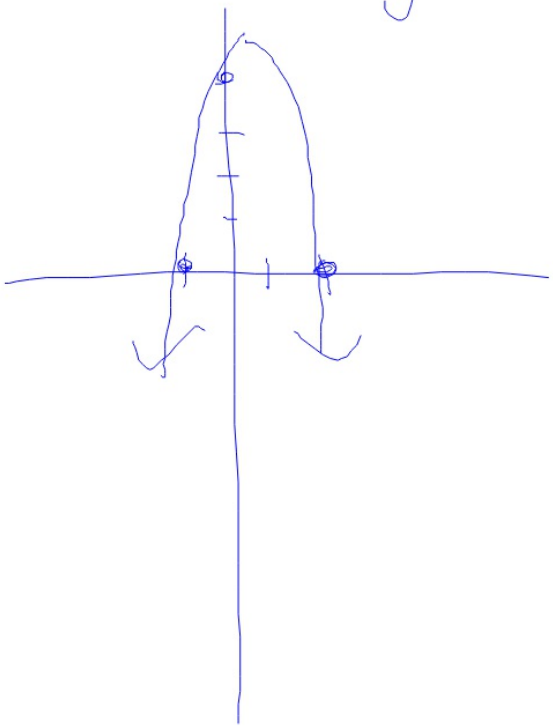
$a = -2$

plug in $X = 0$

$-2(0+1)(0-2)$

$y\text{-int} = 4$

$-1, 2$



Sketch the graph of $f(x) = 2(x-3)^2$ by considering:

a the value of a **b** the y-intercept **c** the x-intercepts.

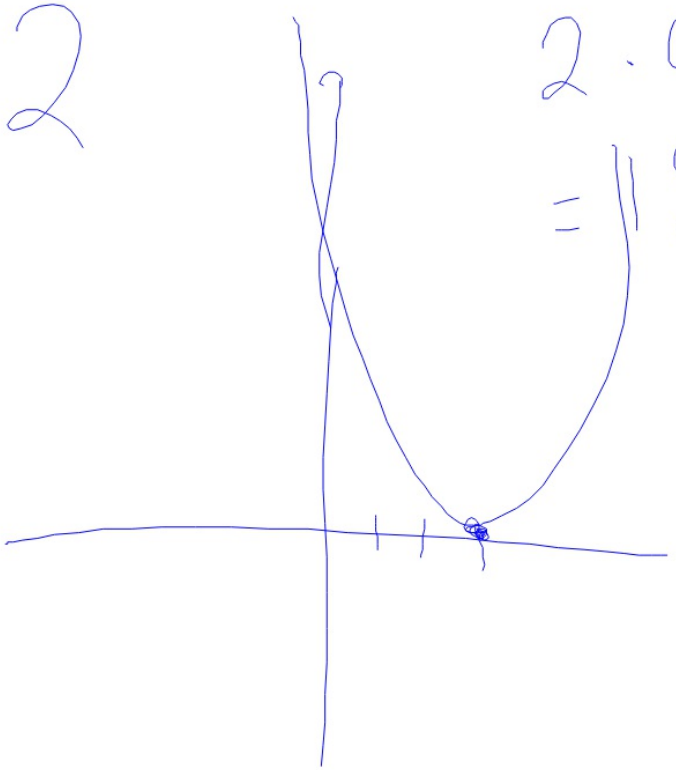
$2(0-3)^2$

$2 \cdot 9$

$= 18$

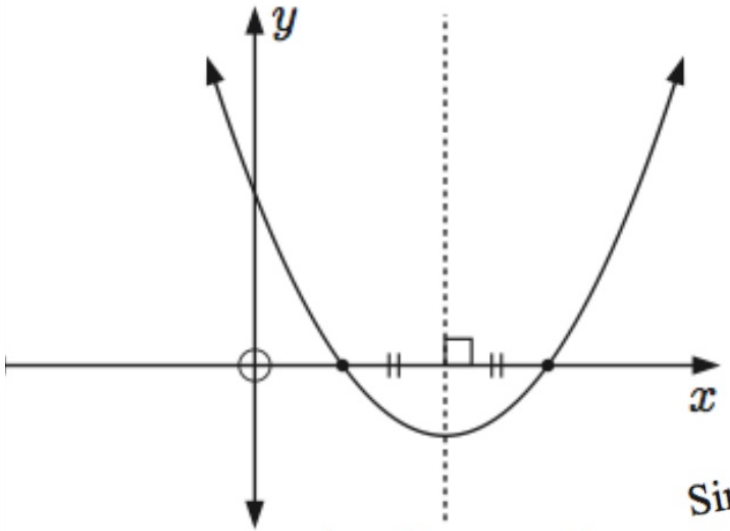
2

3



F AXIS OF SYMMETRY

The graph of any quadratic function is symmetric about a vertical line called the **axis of symmetry**.



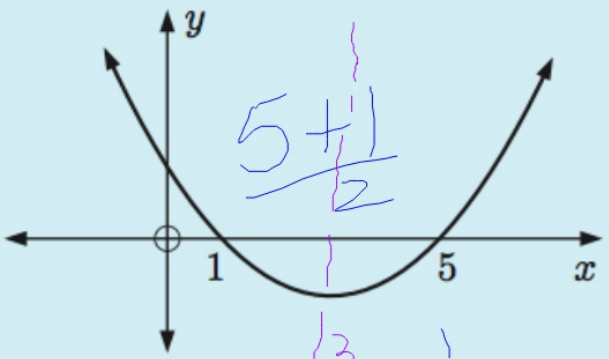
$X =$ (x-value of vertex)

Since the axis of symmetry is vertical, its equation will have the form $x = k$.

If a quadratic function has two x-intercepts, then the axis of symmetry lies halfway between them.

Find the equation of the axis of symmetry for the following quadratic functions:

a

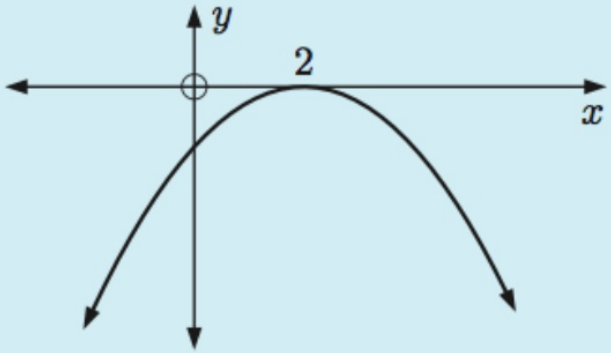


$\frac{5+1}{2}$

3

$3 = x$

b



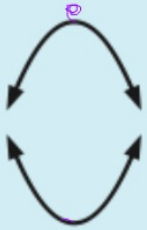
$x = 2$

AXIS OF SYMMETRY OF $y = ax^2 + bx + c$

The equation of the axis of symmetry of $y = ax^2 + bx + c$ is $x = \dfrac{-b}{2a}$.

G VERTEX

The **vertex** or **turning point** of a parabola is the point at which the function has:

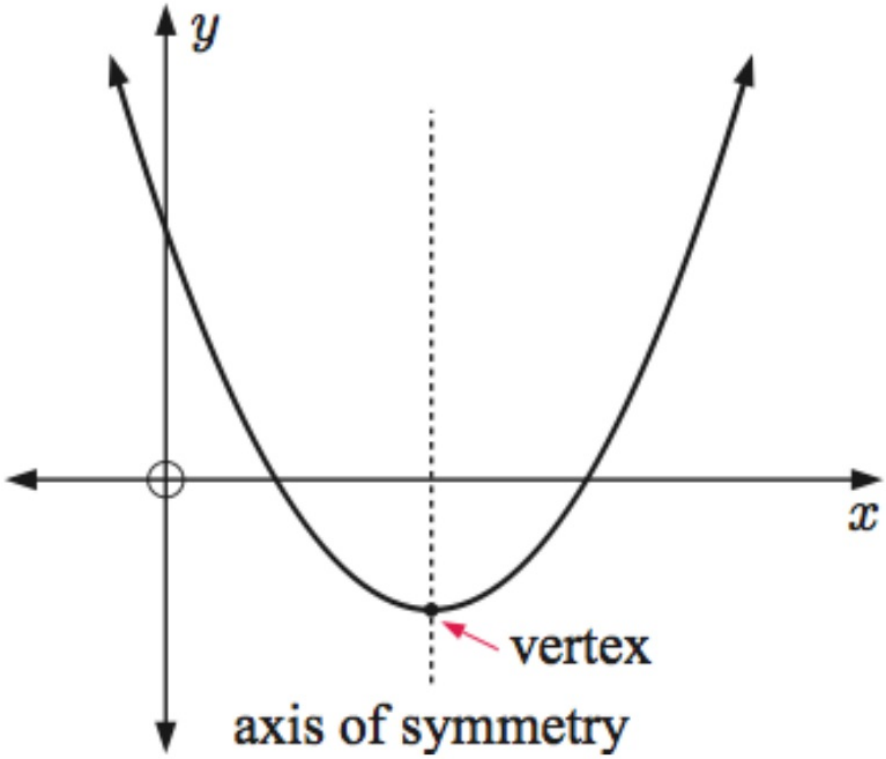
- a **maximum value** for $a < 0$  . We call this point a **local maximum.**

  or

- a **minimum value** for $a > 0$. We call this point a **local minimum.**

The vertex of a quadratic function always lies on the **axis of symmetry**, so the axis of symmetry gives us the x-coordinate of the vertex.

The y-coordinate can be found by substituting this value of x into the function.



Determine the coordinates of the vertex of $f(x) = x^2 + 6x + 4$.

$a=1 \quad b=6 \quad c=4$

$\left(\frac{-b}{2a} , \right)$

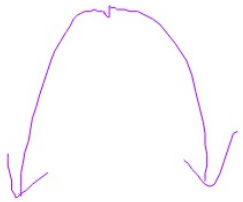
$\frac{-6}{2(1)} = -3$

$(-3, -5)$

$f(-3) = (-3)^2 + 6(-3) + 4$

$9 - 18 + 4$

Use technology to find the vertex of $f(x) = -2x^2 + 2x + 3$.



$(0.5, 3.5)$

Assignment:

Exercise 17 D # 2
E # 2 a-c
F # 1
G # 3 a-c