Chapter 20

Differential calculus

Syllabus reference: 7.1, 7.2, 7.3

Contents:

A

RATES OF CHANGE

A **rate of change** is a measure of how quickly something is changing: moving, increasing, decreasing, growing, etc.

Some examples of rates:

velocity

acceleration

unit price

height vs Age

On a graph, the rate of change is the
Slope ______ of the graph.

gradient

<u>Three ways to find slope:</u>

Graphically: $$m = \frac{rise}{run}$$

Numerically: What's the pattern in the t-chart?

x	y
1	4
2	8
3	12
4	16

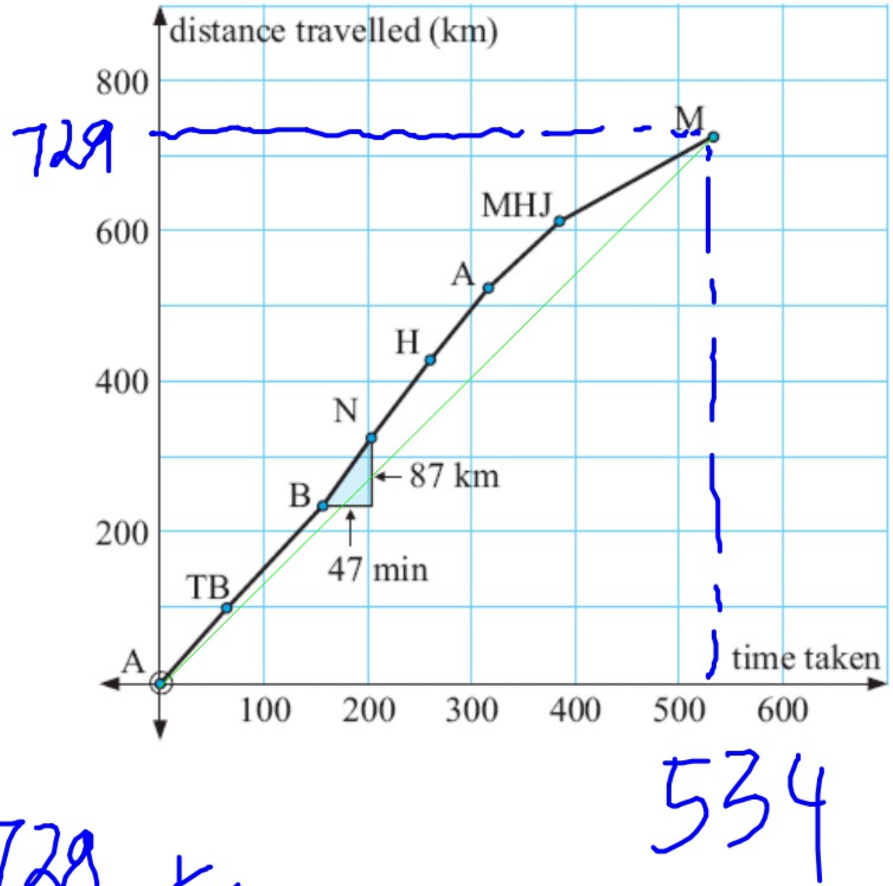
1 1 1

4 4 4

Algebraically: $$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Trip from Adelaide to Melbourne

Place	*Time taken (min)*	*Distance travelled (km)*
Adelaide tollgate	0	0
Tailem Bend	63	98
Bordertown	157	237
Nhill	204	324
Horsham	261	431
Ararat	317	527
Midland H/W Junction	386	616
Melbourne	534	729



Average speed = $\frac{729}{534}$ km/min

Start to finish

Average Rates from Curved Graphs

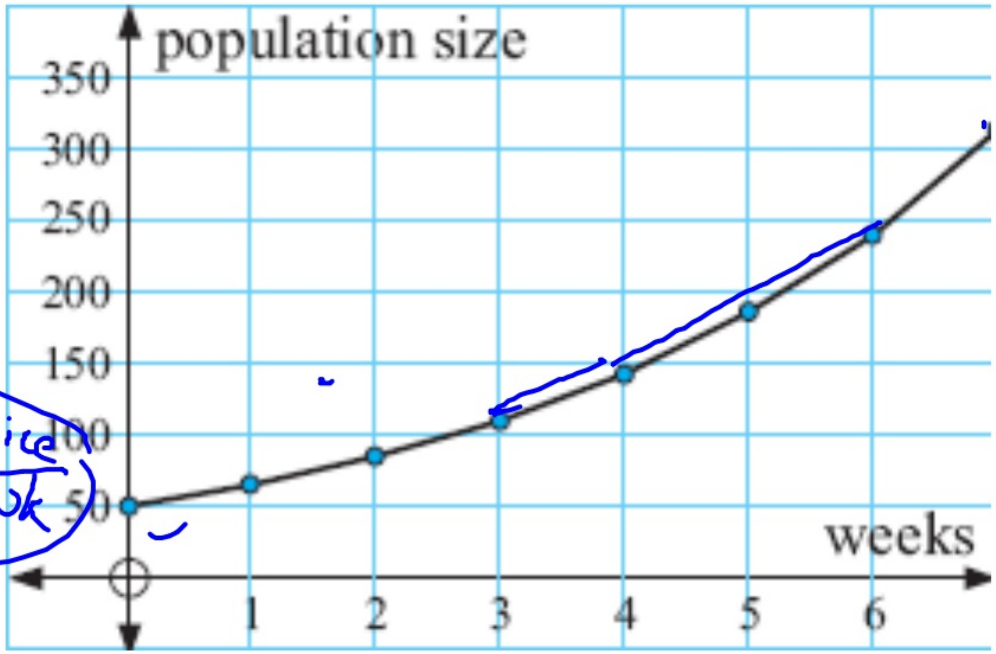
The number of mice in a colony was recorded on a weekly basis.

a Estimate the average rate of increase in population for:

- **i** the period from week 3 to week 6
- **ii** the seven week period.

$$\frac{249-107}{6-3}$$

$$\frac{142}{3} = 47\tfrac{1}{3}\ \frac{\text{mice}}{\text{wk}}$$



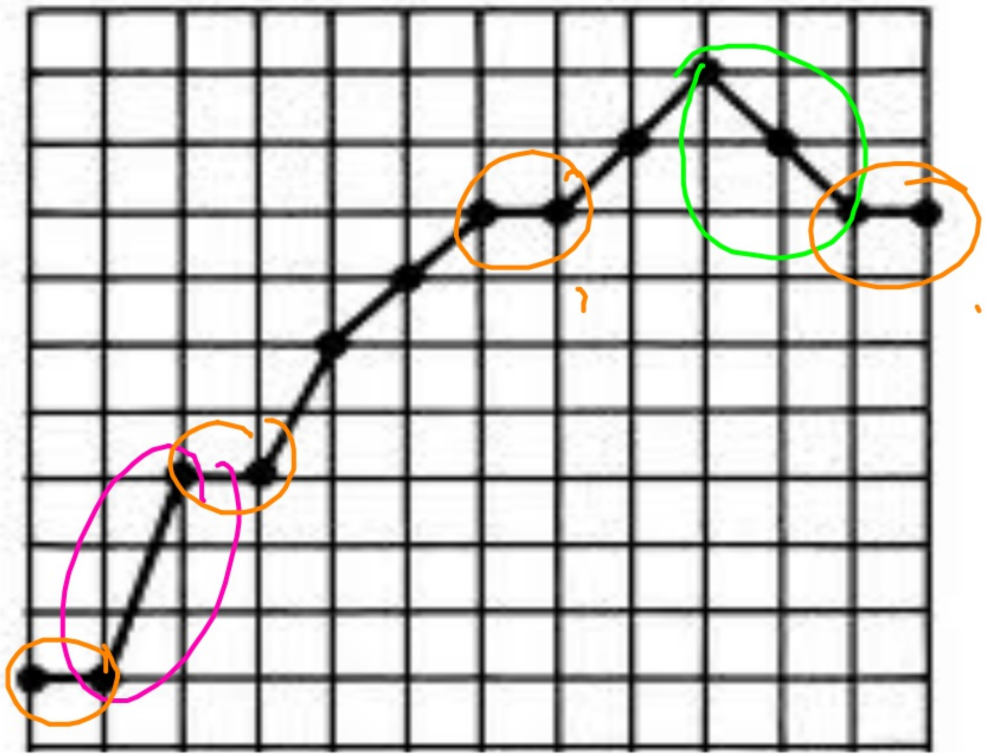
$$\frac{310-50}{7}$$

$$\frac{260}{7}$$

$$37.1\ \frac{\text{mic}}{\text{wk}}$$

b What is the overall trend with regard to population increase over this period?

What is happening to the **slope** of this graph?

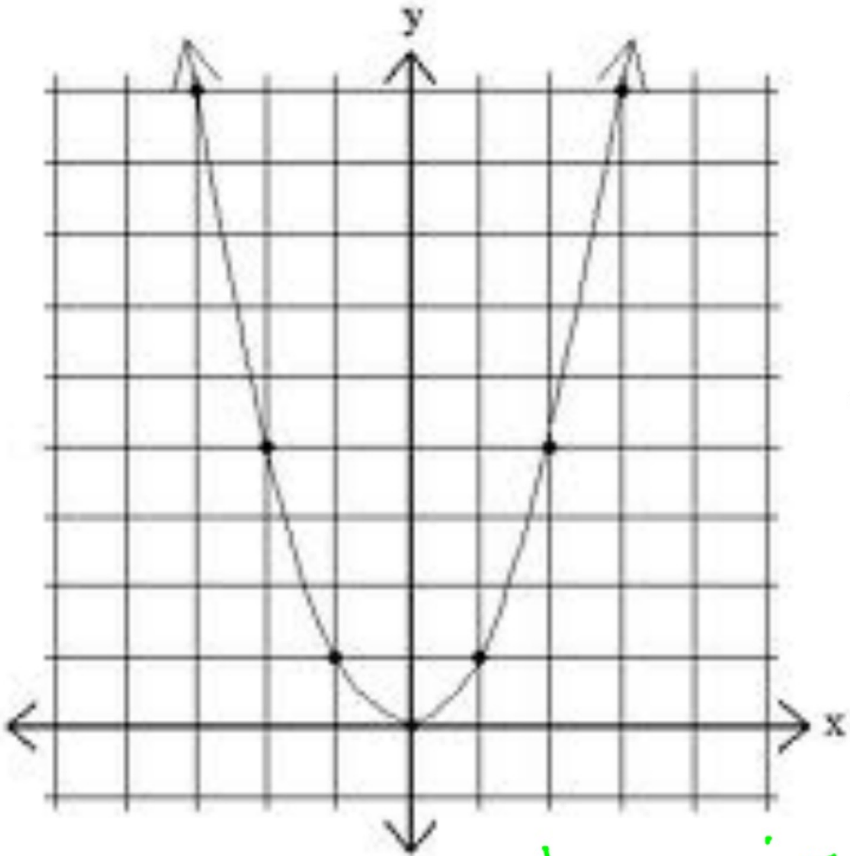


Where is the slope the greatest? $\frac{3}{1}$

Where is the slope the least? -1

Is the slope ever zero or indefined? $\emptyset$

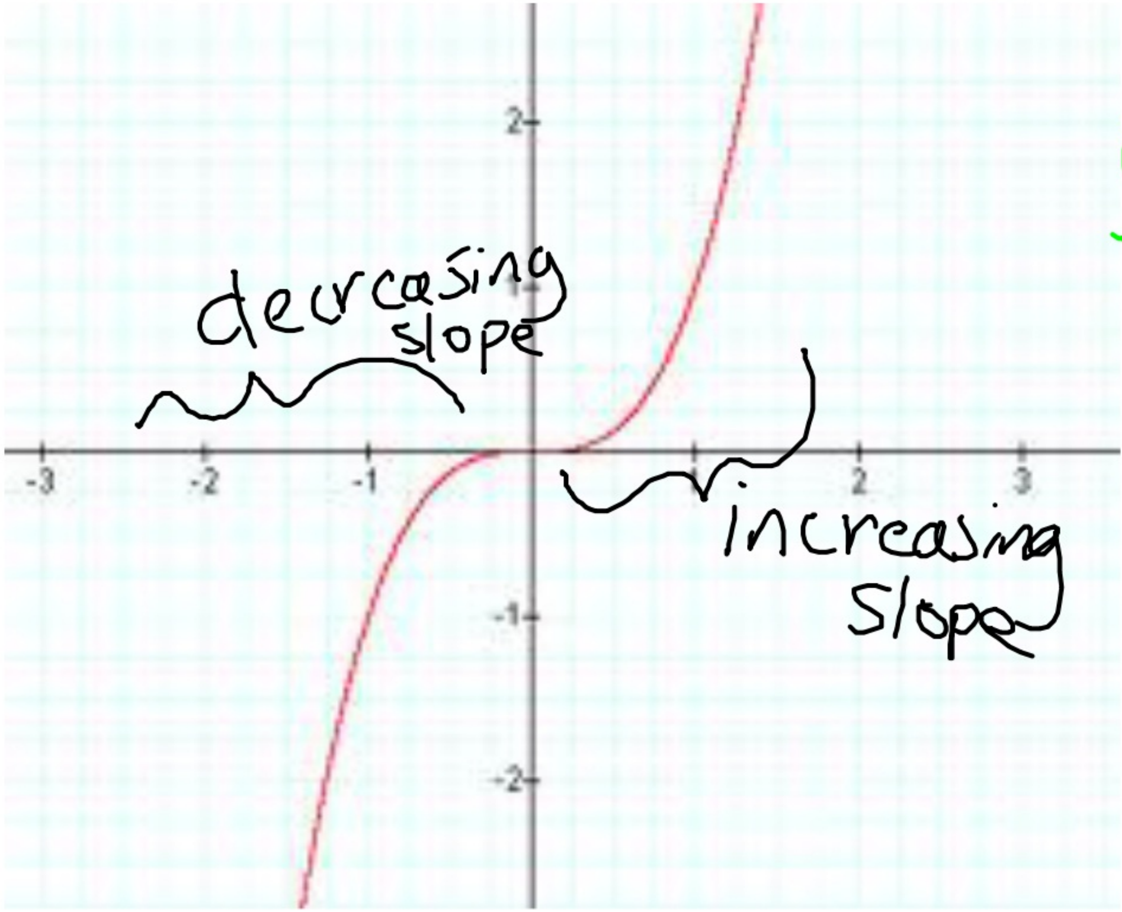
What is happening to the **slope** of this graph?



$y = x^2$

slope is increasing

What is happening to the **slope** of this graph?

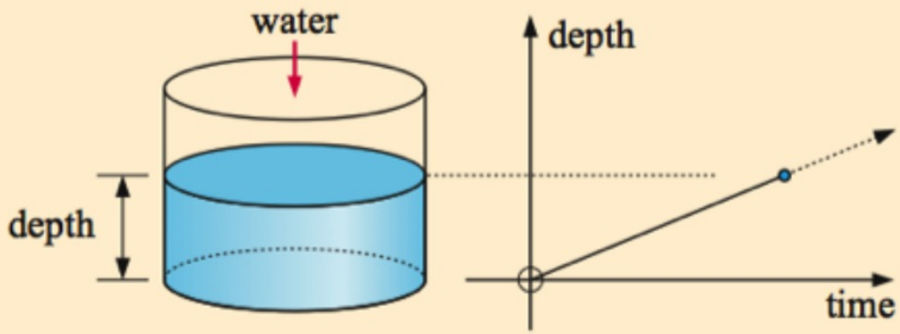


$y = x^3$

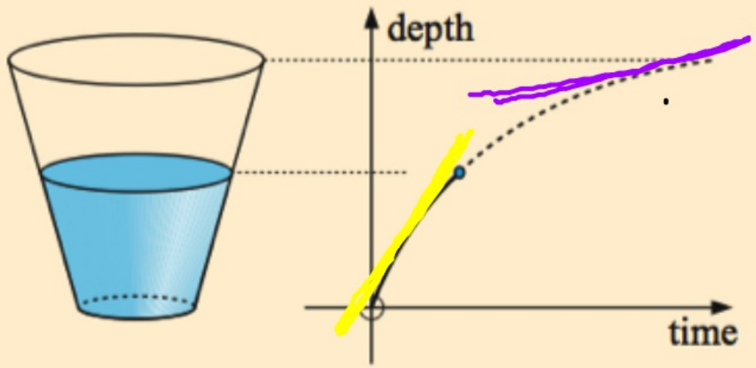
INVESTIGATION 1 CONSTANT AND VARIABLE RATES OF CHANGE

When water is added at a **constant rate** to a cylindrical container, the depth of water in the container is a linear function of time.

The depth-time graph for a cylindrical container is shown alongside.



In this investigation we explore the changes in the graph for different shaped containers such as a conical vase.

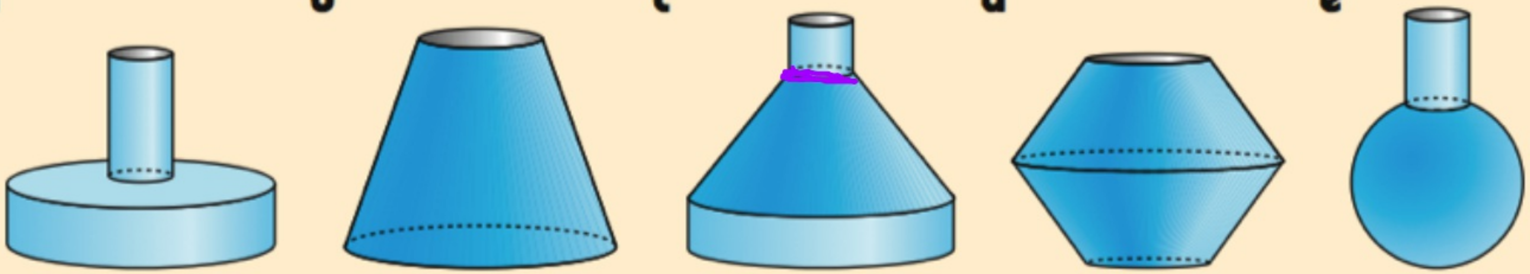


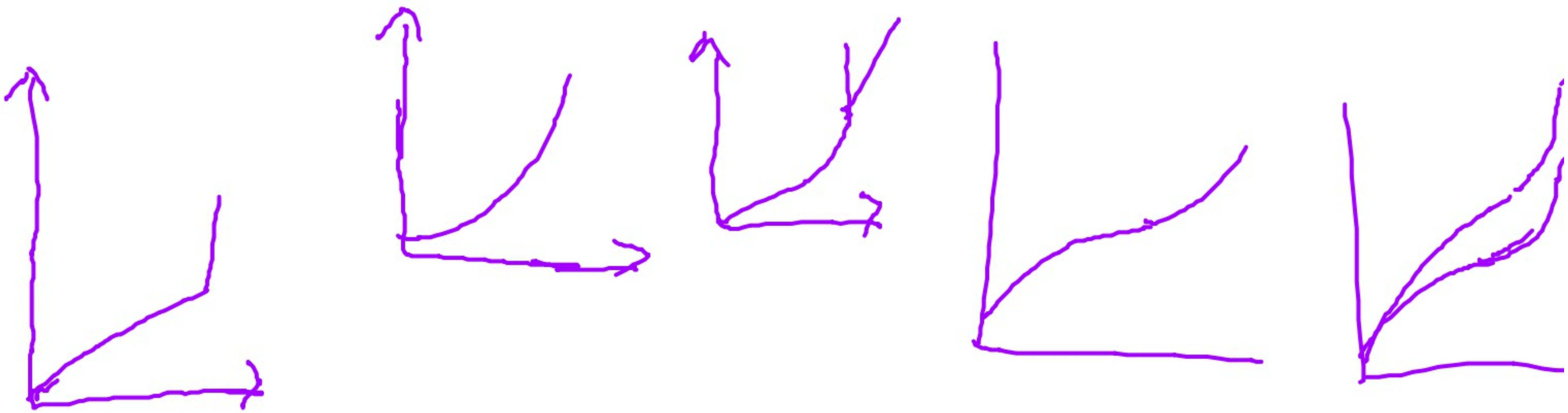
What to do:

1. What features of the graph indicate a rate of change in water level that is:
   a constant **b** variable?

2. For each of the following containers, draw a depth-time graph as water is added:

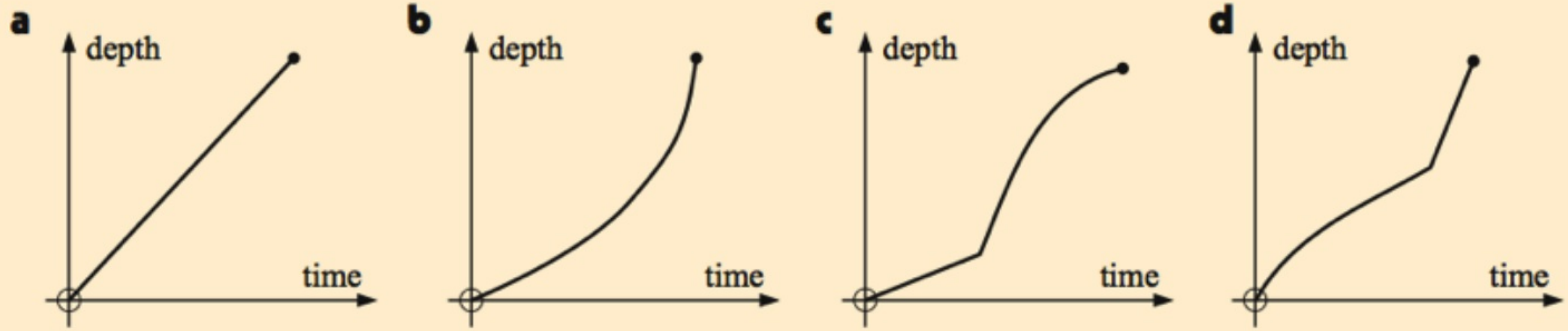
a **b** **c** **d** **e**

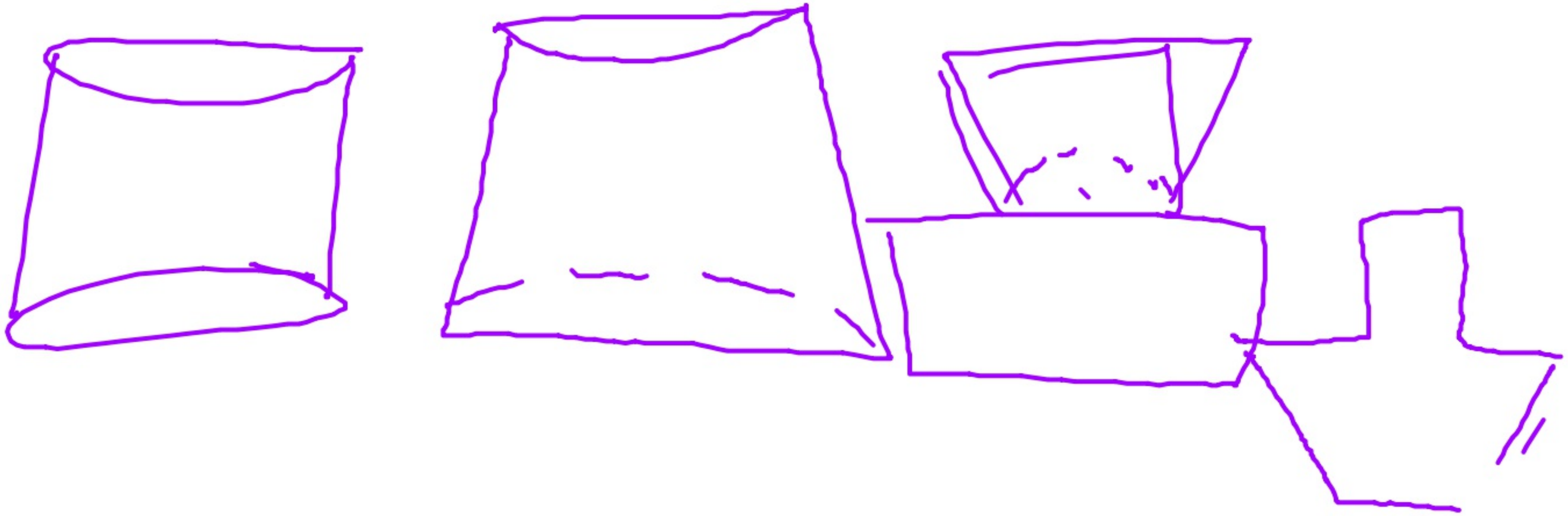




3 Write a brief report on the connection between the shape of a vessel and the shape of its depth-time graph. You may wish to discuss this in parts. For example, first examine cylindrical containers, then conical, then other shapes.

4 Suggest possible container shapes that will have the following depth-time graphs:





Assignment:

20 A.1 # 3, 5

20 A.2 # 1, 2