Chapter 20

Differential calculus

Syllabus reference: 7.1, 7.2, 7.3

Contents:

3h 56min = 3.93 h

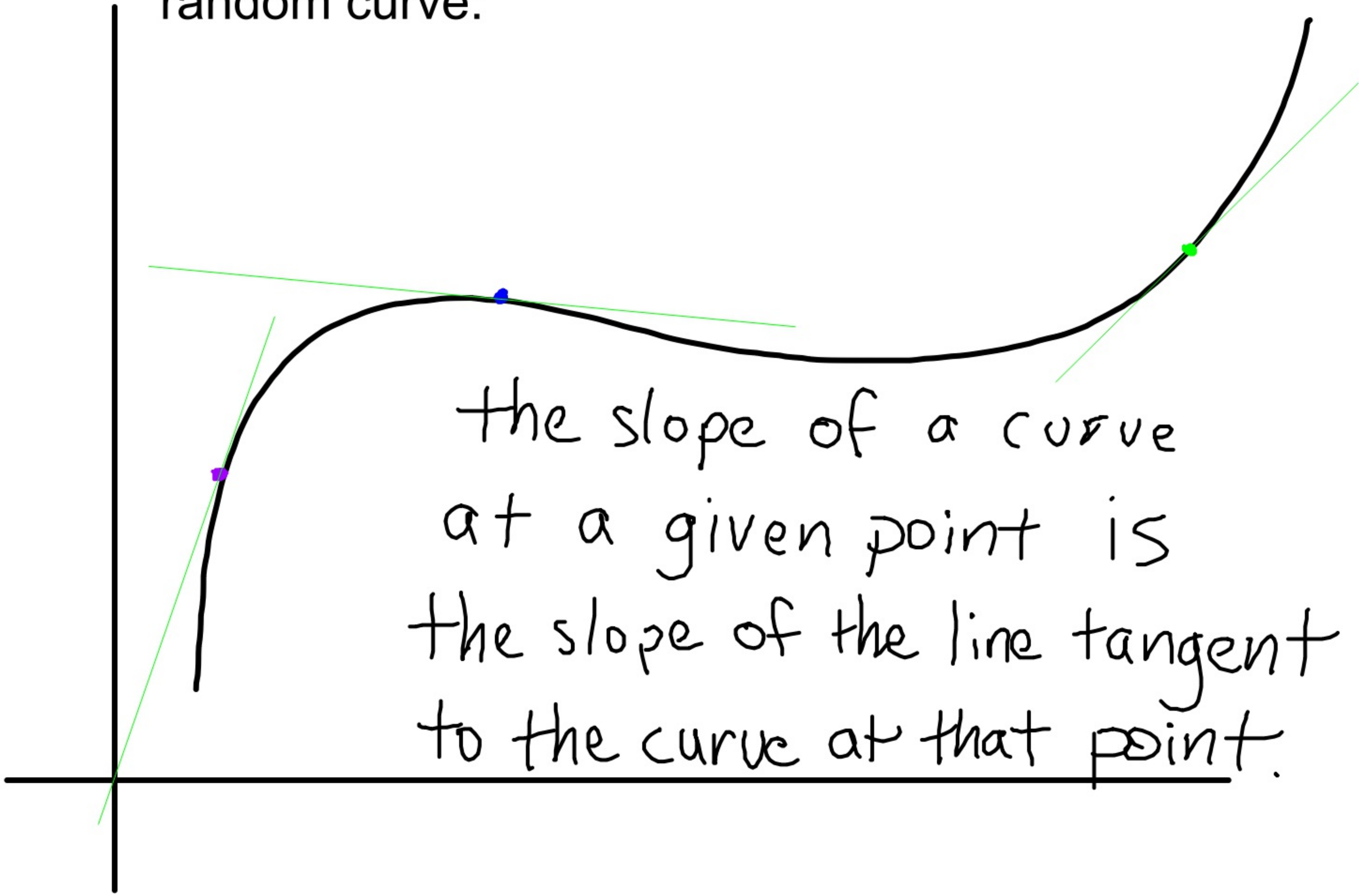
$$\frac{350}{3.93}$$

$$\frac{35000}{}$$

B INSTANTANEOUS RATES OF CHANGE

A moving object, such as a car, an aeroplane, or a runner has a variable speed. At a particular instant in time, the speed of the object is called its **instantaneous speed**.

Let's look at the rate of change for a random curve:

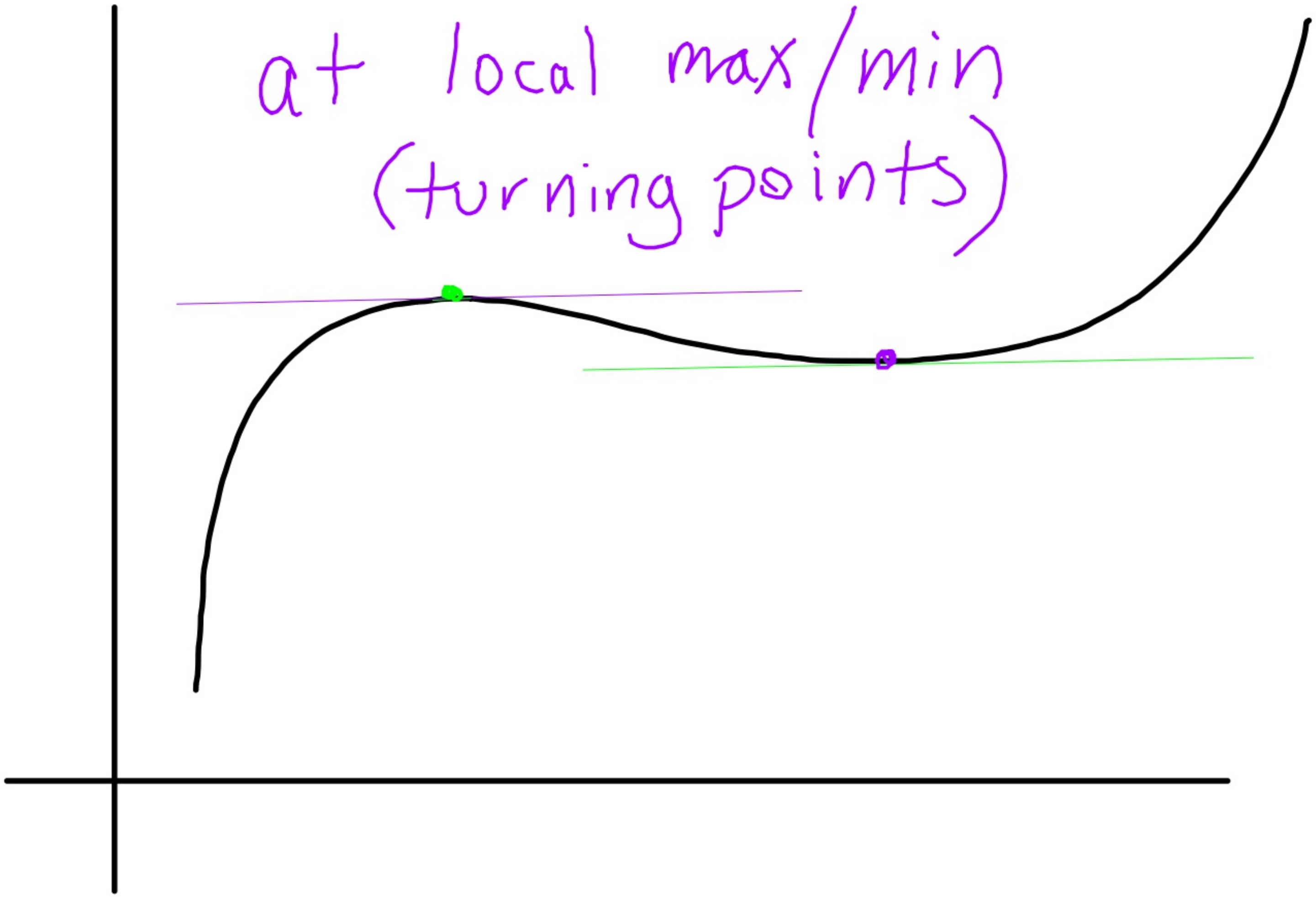


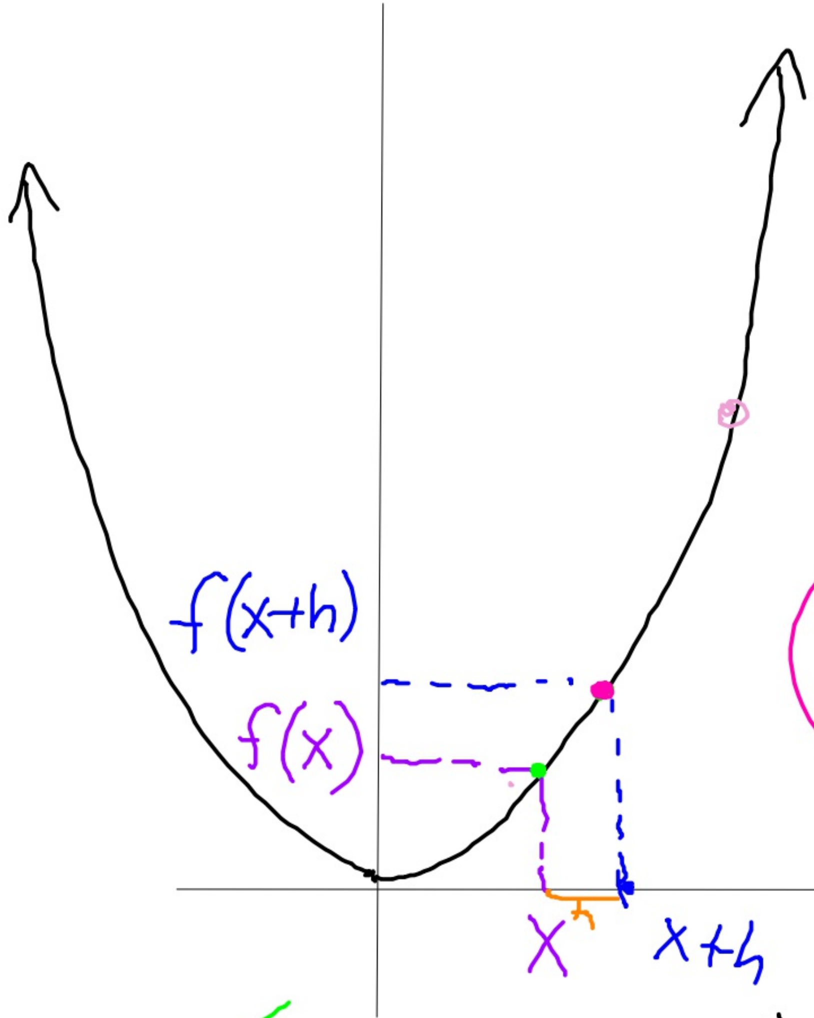
the slope of a curve at a given point is the slope of the line tangent to the curve at that point.

Let's look at the rate of change for a random curve:

Where is the slope = 0?

at local max/min (turning points)





slope of the line between these points

$$\frac{f(x+h)-f(x)}{\cancel{x}+h-\cancel{x}}$$

$(x+h,\ f(x+h))$

$(x,\ f(x))$

As h gets smaller the slope between the points approaches the slope at $(x, f(x))$

THE ALGEBRAIC METHOD

For finding the slope of a curve at a given point

(this is also the slope of the line tangent at that point)

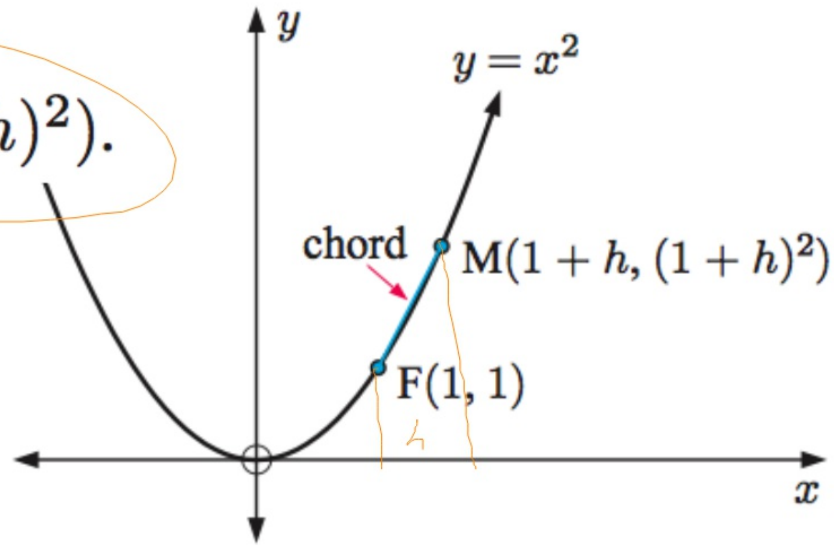
consider the curve $y = x^2$ and the tangent at F(1, 1).

Let the moving point M have x-coordinate $1 + h$, where $h \neq 0$.

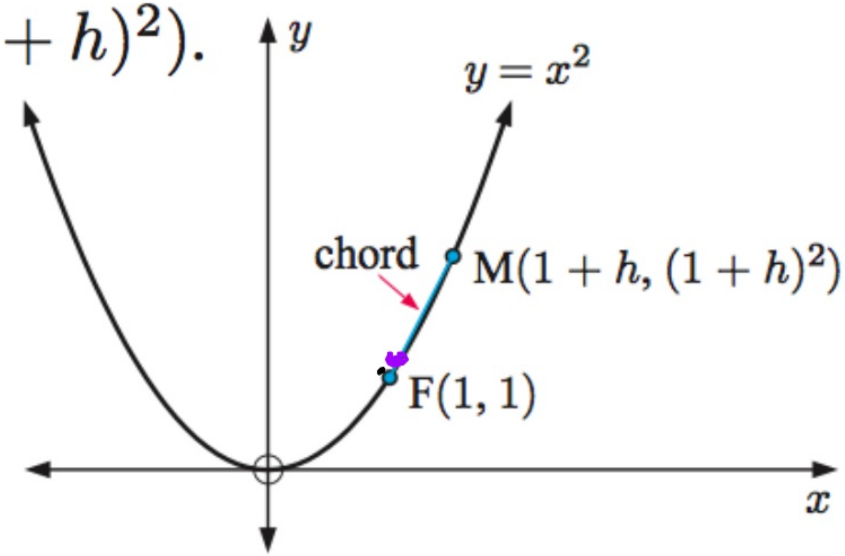
So, M is at $(1 + h, (1 + h)^2)$.

$X = 1 + h$

$X^2 = (1 + h)^2$



So, M is at $(1+h, (1+h)^2)$.



$(1+h)(1+h)$

The gradient of chord FM is

$$\frac{(1+h)^2-1}{1+h-1} = \frac{\cancel{1}+2h+h^2-\cancel{1}}{h} = \frac{\cancel{h}(2+h)}{\cancel{h}}$$

$$= 2+h$$

zero

slope @ $(1,1) = 2$

Use the algebraic method to find the gradient of the tangent to $y = x^2$ at the point where $x = 2$.

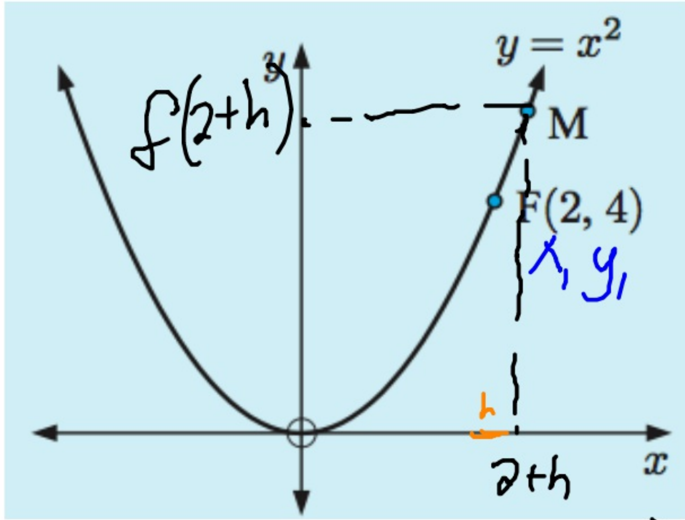
$(2+h)(2+h)$

$$\frac{(2+h)^2 - 4}{\cancel{2}+h-\cancel{2}}$$

$$\frac{\cancel{4}+4h+h^2-\cancel{4}}{h}$$

$$\frac{\cancel{h}(4+h)}{\cancel{h}} = 4+h$$

$$\lim_{h \to 0} = 4$$



$$M(\underbrace{2+h}_{x_2}, \underbrace{f(2+h)}_{y_2})$$

$$(2+h, (2+h)^2)$$

The slope of a line tangent to the graph at *a* is

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

This is the derivative of the function.

*** Congratulations, you've just learned some calculus.*

Notation for derivatives:

original function	derivative	
$f(x)$	$f'(x)$	
y	y'	
	$\dfrac{dy}{dx}$	$\dfrac{\Delta y}{\Delta x}$

Assignment / Example

Long-Hand Derivatives

Find f '(x) of the function.

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

1) $f(x) = 5x - 1$

$$\frac{5(x+h) - 1 - (5x - 1)}{h}$$

$f(a+h)$

$5(x+h) - 1$

plug $(x+h)$ in for x

$$\frac{\cancel{5x} + 5h \cancel{-1} - \cancel{5x} \cancel{+1}}{h} = \frac{5\cancel{h}}{\cancel{h}} = 5$$

Assignment / Example

Find f '(x) of the function.

2) $f(x) = x^2$

3) $f(x) = 4x + 7$

4) $f(x) = x^2 + 3x$

Assignment / Example

Find f '(x) of the function.

3) $$f\left(x\right) = 4x + 7$$

Assignment / Example

Find f '(x) of the function.

4) $f\left(x\right)=x^2+3x$

Assignment / Example

Find f '(x) of the function.

5) $$f(x) = 3x^3$$

Assignment: Long-Hand Derivatives

Find f '(x) of each function.

1) $f\left(x\right)=5x-1$

2) $f\left(x\right)=x^2$

3) $f\left(x\right)=4x+7$

4) $f\left(x\right)=x^2+3x$

5) $f\left(x\right)=3x^3$

6) $f\left(x\right)=\dfrac{1}{x}$

7) $f(x)=3x+2$

8) $f(x)=-2x^2+x-1$

9) $f(x)=1-x^2$

10) $f(x)=x^3$

Assignment: Long-Hand Derivatives

Find f '(x) of each function.

1) $f(x) = 5x - 1$

2) $f(x) = x^2$

3) $f(x) = 4x + 7$

4) $f(x) = x^2 + 3x$

5) $f(x) = 3x^3$

6) $f(x) = \frac{1}{x}$

7) $f(x) = 3x + 2$

8) $f(x) = -2x^2 + x - 1$

9) $f(x) = 1 - x^2$

10) $f(x) = x^3$