# Chapter 21

# Applications of differential calculus

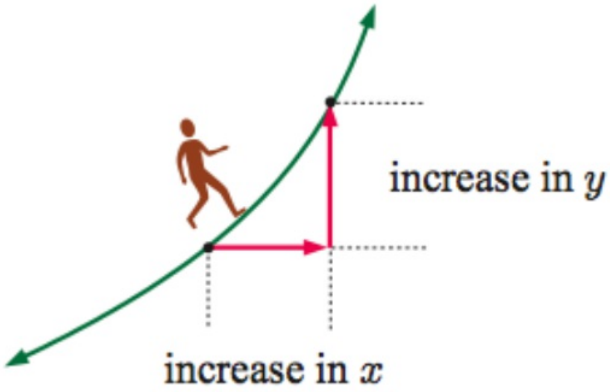
**Syllabus reference: 7.4, 7.5, 7.6**

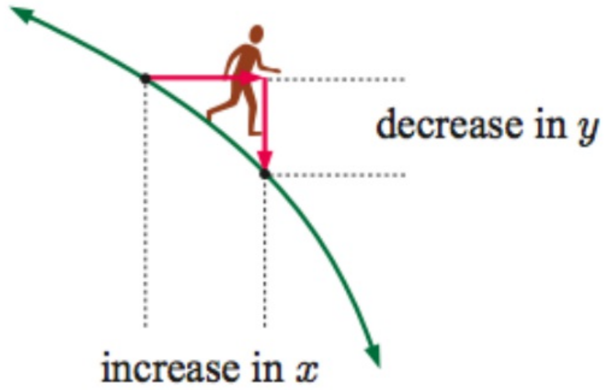
**Contents:**

# A INCREASING AND DECREASING FUNCTIONS

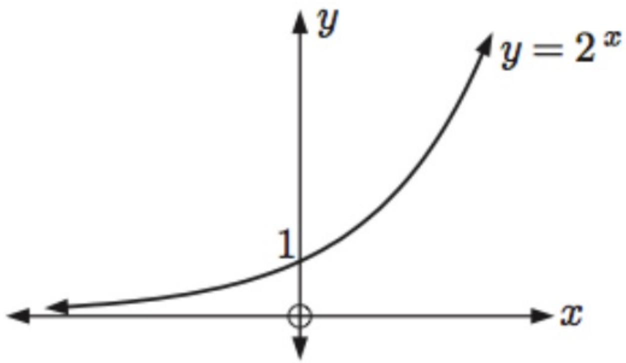
On an interval where the function is **increasing**, an increase in $x$ produces an **increase** in $y$.

On an interval where the function is **decreasing**, an increase in $x$ produces a **decrease** in $y$.



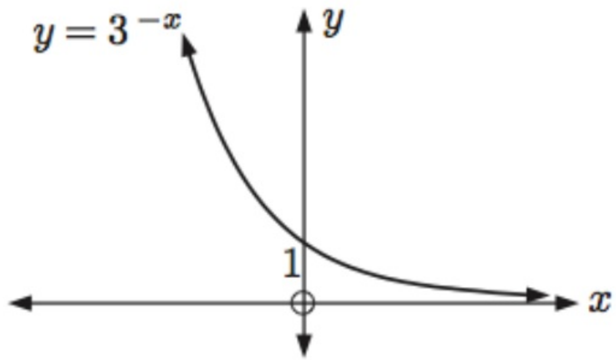



For example:



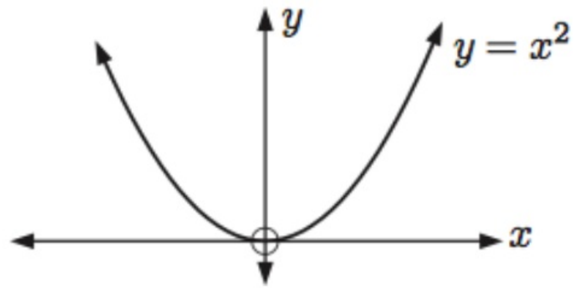

$y = 2^x$ is increasing for all $x$.

We say $y = 2^x$ is an **increasing function**.




$y = 3^{-x}$ is decreasing for all $x$.

We say $y = 3^{-x}$ is a **decreasing function**.



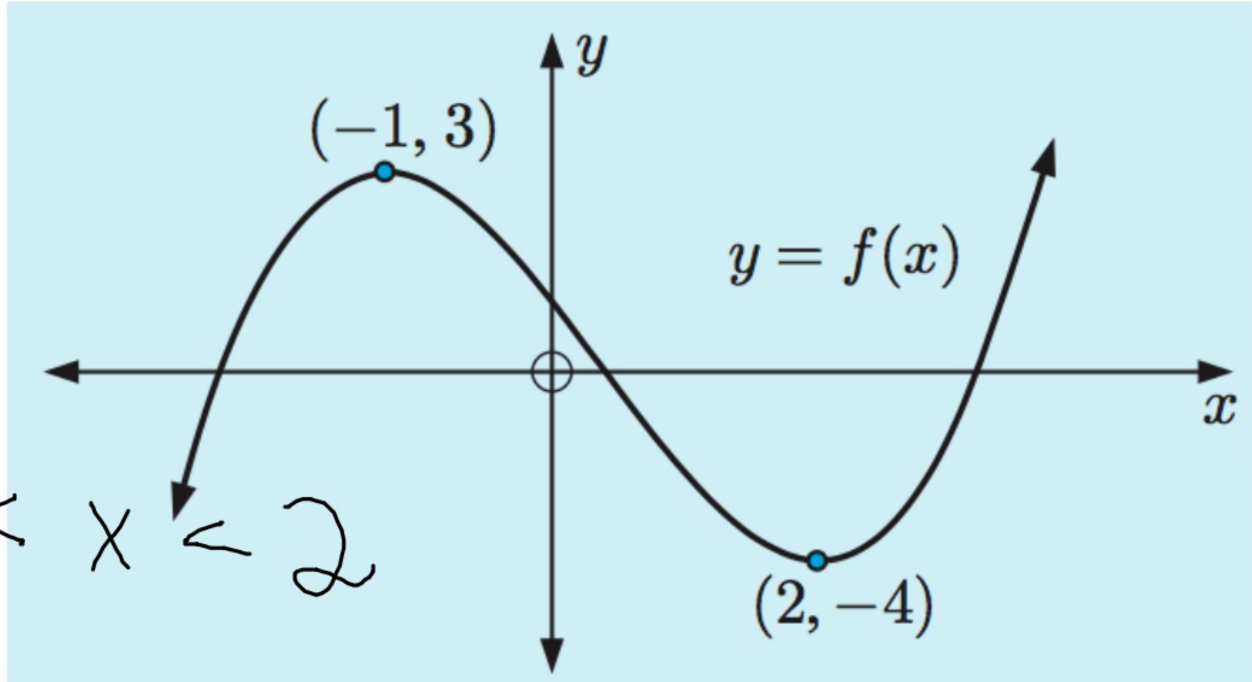

$y = x^2$ is decreasing for $x \leqslant 0$ and increasing for $x \geqslant 0$.

Find intervals where $f(x)$ is:

**a** increasing

**b** decreasing.

a) $x < -1$ and $2 < x$




b) $-1 < x < 2$

We can deduce when a curve is increasing or decreasing by considering $f'(x)$ on the interval in question.

For the functions that we deal with in this course:

- $f(x)$ is **increasing** on $S$ if $f'(x) \geqslant 0$ for all $x$ in $S$.
- $f(x)$ is **strictly increasing** if $f'(x) > 0$ for all $x$ in $S$.
- $f(x)$ is **decreasing** on $S$ if $f'(x) \leqslant 0$ for all $x$ in $S$.
- $f(x)$ is **strictly decreasing** if $f'(x) < 0$ for all $x$ in $S$.

# SIGN DIAGRAMS


Sign diagrams for the derivative illustrate the intervals where a function is increasing or decreasing.

The critical values for f'(x) are the values of x for which f'(x) = 0 or f'(x) is undefined.

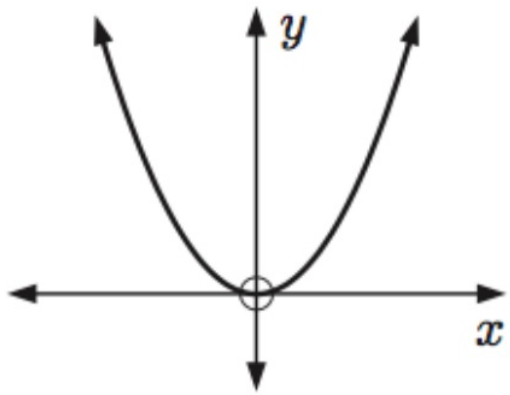
When f'(x) = 0, the critical values are shown wiht a tick mark on the sign diagram.

When f'(x) is undefined, the critical values are shown with a vertical dotted line of the sign diagram.

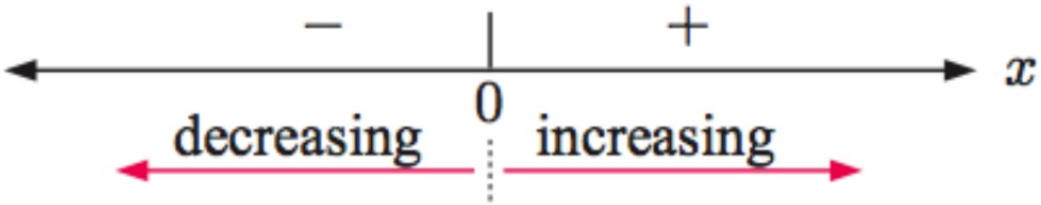
The sign diagram indicates whether the derivative is negative or positive by using a + or -.

Consider the following examples:

- $f(x) = x^2$

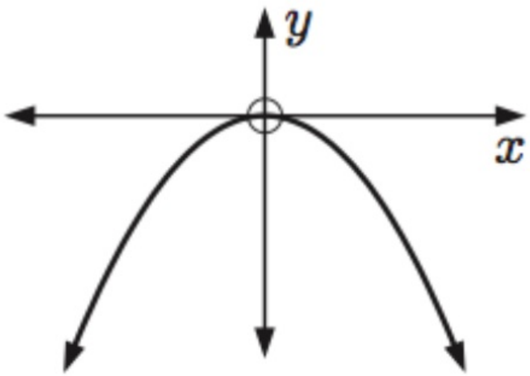



$f'(x) = 2x$ which has sign diagram

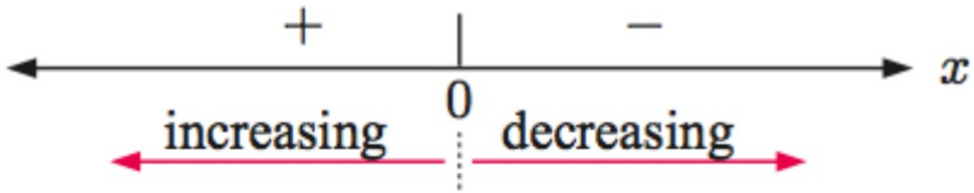



$\therefore$ $f(x) = x^2$ is decreasing for $x \leqslant 0$
and increasing for $x \geqslant 0$.

- $f(x) = -x^2$

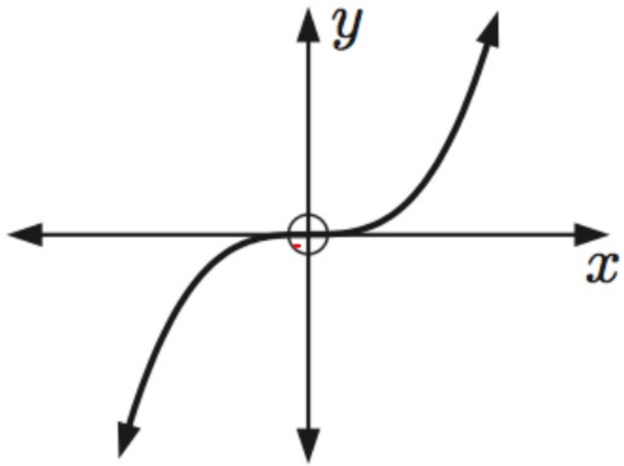



$f'(x) = -2x$ which has sign diagram

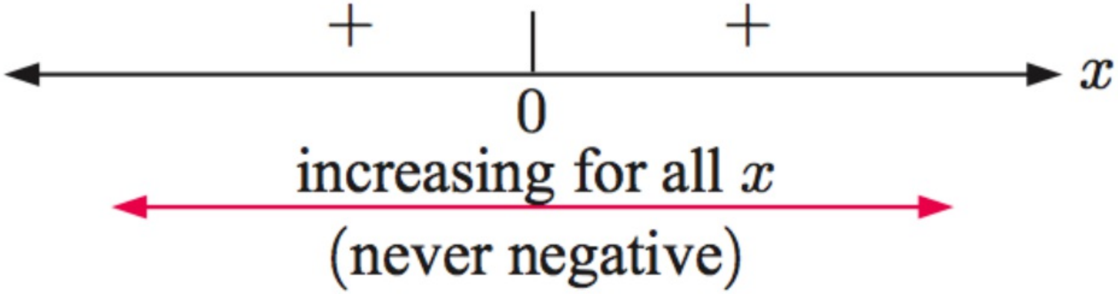



$\therefore$ $f(x) = -x^2$ is increasing for $x \leqslant 0$
and decreasing for $x \geqslant 0$.

- $f(x) = x^3$

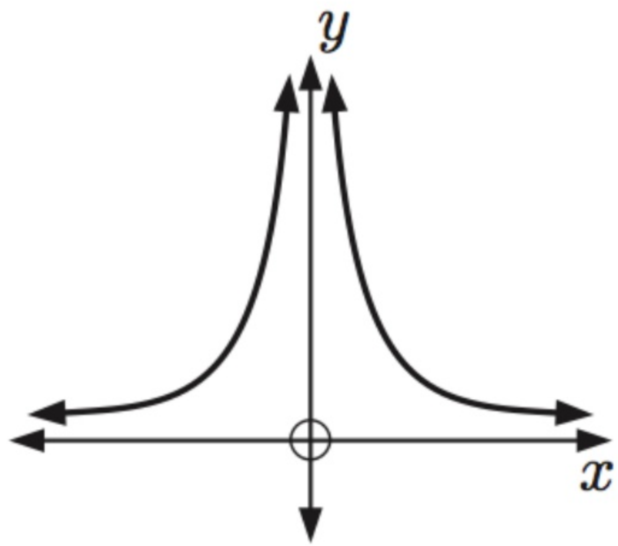



$f'(x) = 3x^2$ which has sign diagram

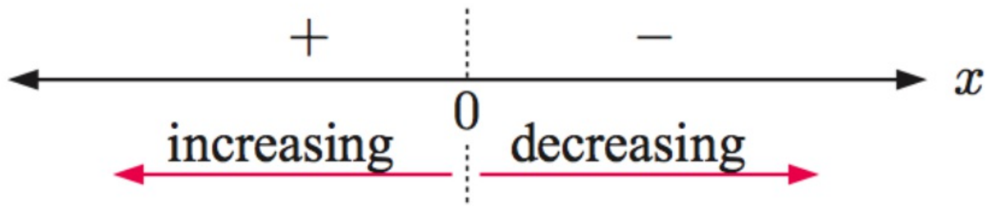



$\therefore \quad f(x) = x^3$ is increasing for all $x$.

- $f(x) = \dfrac{1}{x^2}$




$f'(x) = -2x^{-3} = -\dfrac{2}{x^3}$ which has sign diagram




$\therefore \quad f(x) = \dfrac{1}{x^2}$ is increasing for $x < 0$

and decreasing for $x > 0$.

How do we find the critical values?

When is the slope equal to zero or undefined?

set $f'(x) = \emptyset$

put these values into the sign diagram.

Find the intervals where $f(x) = 2x^3 + 3x^2 - 12x - 5$ is increasing or decreasing.

$$f'(x) = 6x^2 + 6x - 12$$

$$0 = 6(x^2 + x - 2)$$

$$0 = 6(x-1)(x+2)$$

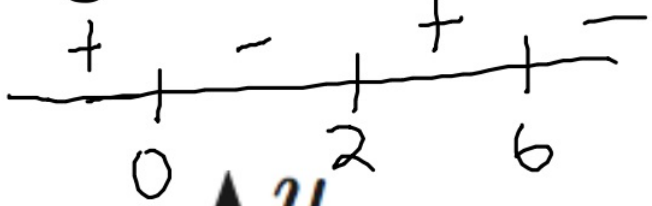
$$1 \qquad -2$$

Assignment:

# Exercise 21 A all

Draw the sign diagram of:

a) f'(x)




b) f (x)