C RATES OF CHANGE

TIME RATES OF CHANGE

There are countless quantities in the real world that vary with time.

For example:
- temperature varies continuously
- the height of a tree increases as it grows
- the prices of stocks and shares vary with each day's trading.

Varying quantities can be modelled using functions of time.

For example:

- Suppose $s(t)$ models the distance travelled by a runner.

  $\frac{ds}{dt}$ or $s'(t)$ is the instantaneous *speed* of the runner.

  It might have units metres per second or $\text{m}\,\text{s}^{-1}$.

** pay close attention to the *units* of the rate of change **

The volume of air in a hot air balloon after t minutes is given by $V = 2t^3 - 3t^2 + 10t + 2$ m^3 where $0 \leqslant t \leqslant 8$.

Find:

a the initial volume of air in the balloon $2\,m^3$

b the volume when $t = 8$ minutes

$2(8)^3 - 3(8)^2 + 10(8) + 2 = 914\,m^3$

c $\dfrac{dV}{dt}$

d the rate of increase in volume when $t = 4$ minutes.

c) $\dfrac{dV}{dt} = 6t^2 - 6t + 10$

d) $6(4)^2 - 6(4) + 10$

$= 82\,m^3$

You try it: 21 C.1

4 Water is draining from a swimming pool such that the remaining volume of water after t minutes is $V = 2(50-t)^2$ m^3. Find:

a the average rate at which the water leaves the pool in the first 5 minutes

b the instantaneous rate at which the water is leaving at $t = 5$ minutes.

0 - 5 min

$V_0 = 5000\ m^3$

$\frac{5000-4050}{5}$

$V_5 = 4050\ m^3$

$\frac{950}{5} = 190\ \frac{m^3}{min}$

6 The height of a palm tree is given by $H = 20 - \frac{18}{t}$ metres, where t is the number of years after the tree was planted from an established potted juvenile tree, and $t \geqslant 1$.

a How high was the palm after 1 year?

b Find the height of the palm at $t = 2, 3, 5, 10,$ and 50 years.

c Find $\frac{dH}{dt}$ and state its units.

d At what rate is the tree growing at $t = 1, 3,$ and 10 years?

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c Find $\frac{dH}{dt}$ and state its units.

d At what rate is the tree growing at $t = 1, 3$, and 10 years?

e Explain why $\frac{dH}{dt} > 0$ for all $t \geqslant 1$. What does this mean in terms of the tree's growth?

The cost of producing x items in a factory each day is given by

$$C(x) = \underbrace{0.000\,13x^3 + 0.002x^2}_{\text{cost of labour}} + \underset{\substack{\uparrow \\ \text{raw material costs}}}{5x} + \underset{\substack{\uparrow \\ \text{fixed or overhead costs such as heating, cooling, maintenance, rent}}}{2200}$$

a Find $C'(x)$.

b Find $C'(150)$. Interpret this result.

c Find $C(151) - C(150)$. Compare this with the answer in **b**.

You try it: 21 C.2

4 Seablue make denim jeans. The cost model for making x pairs per day is

$$C(x) = 0.0003x^3 + 0.02x^2 + 4x + 2250 \text{ dollars.}$$

a Find $C'(x)$.

b Find $C'(220)$. What does it estimate?

c Find $C(221) - C(220)$. What does this represent?

8 The profit made by selling x items is given by $P(x) = 5x - 2000 - \frac{x^2}{10\,000}$ dollars.

a Graph $P(x)$ using technology.

b Determine the sales levels which produce a profit.

c Find $P'(x)$.

d Hence find x such that the profit is increasing.