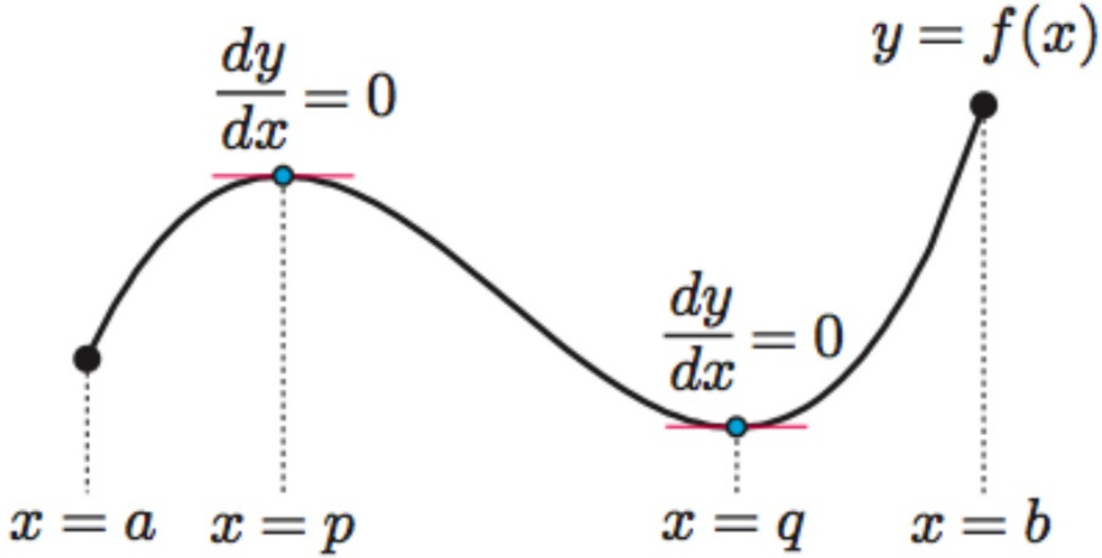
## D OPTIMISATION

The process for finding the maximum or minimum value for a function is called optimization, and the solution is called the optimum.

** remember: the max or min value does not always occur when the derivative is zero. You must also examine the endpoints of the domain in case there's a global max or min. **

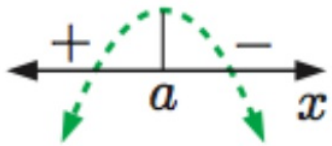



max: $x = b$

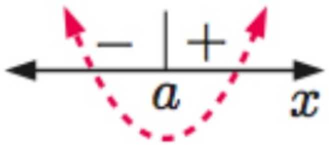
min: $x = q$

When you find an optimum solution, you must have a test to verify that it is a max or min.

## 1) SIGN DIAGRAM TEST

- 

then we have a **local maximum**

- 

then we have a **local minimum.**

## 2) GRAPHICAL TEST

-  then we have a **local maximum**

-  then we have a **local minimum.**

## OPTIMISATION PROBLEM SOLVING METHOD

*Step 1:* If necessary, draw a large, clear diagram of the situation.

*Step 2:* Construct a formula with the variable to be optimised as the subject. It should be written in terms of a single variable such as $x$. You should write down what restrictions there are on $x$.

*Step 3:* Find the **first derivative** and find the values of $x$ where it is **zero**.

*Step 4:* If there is a restricted domain such as $a \leqslant x \leqslant b$, the maximum or minimum may occur either when the derivative is zero or else at an endpoint.

Show using the **sign diagram test** or the **graphical test**, that you have a maximum or a minimum.

A 4 litre container must have a square base, vertical sides, and an open top. Find the most economical shape which minimises the surface area of material needed.

$1\,cm^3 = 1\,mL$

$4\,L = 4000\,mL = 4000\,cm^3$

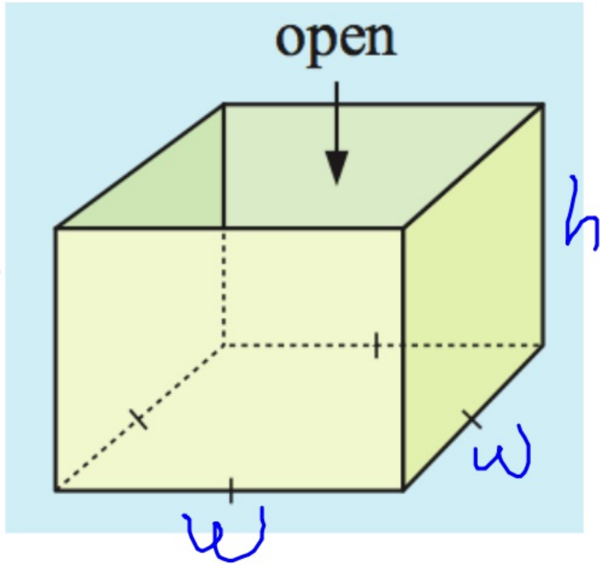
$SA = w^2 + 4hw$

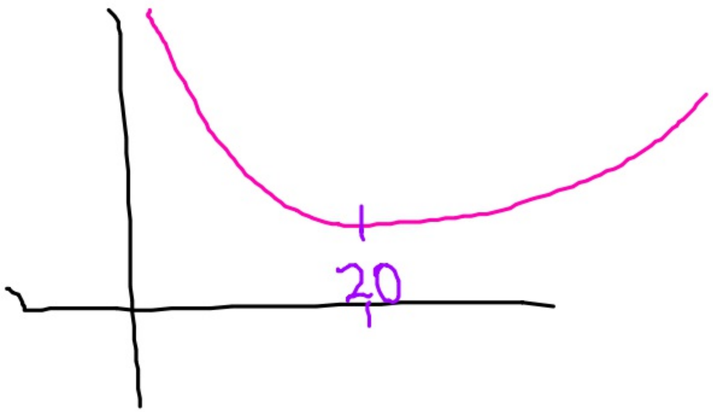
$4000 = w^2 h$

$h = \frac{4000}{w^2}$

$SA = w^2 + 4\left(\frac{4000}{w^2}\right)w$

$SA = w^2 + \frac{16000}{w}$



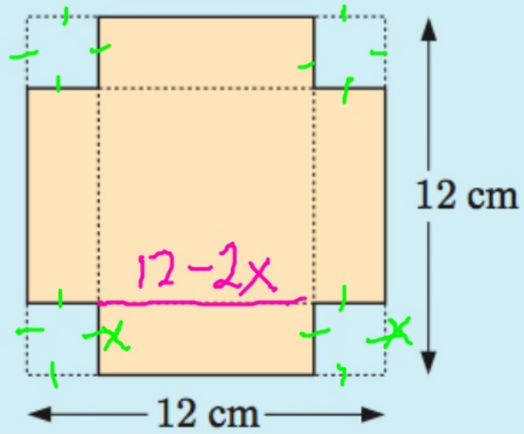

$w = 20\,cm$

$4000 = 20^2 h$

$h = 10\,cm$

A square sheet of metal 12 cm × 12 cm has smaller squares cut from its corners as shown.

What sized square should be cut out so that when the sheet is bent into an open box it will hold the maximum amount of liquid?




$$V = 4x^3 - 48x^2 + 144x$$

$$\frac{dV}{dx} = 12x^2 - 96x + 144$$

$$12(x^2 - 8x + 12)$$

$$(x-6)(x-2)$$

~~$x = 6$~~

$x = 2$

## You try it: 21 D.1

**4** A manufacturer can produce $x$ fittings per day where $0 \leqslant x \leqslant 10\,000$. The production costs are:

- €1000 per day for the workers
- €2 per day per fitting
- €$\frac{5000}{x}$ per day for running costs and maintenance.

How many fittings should be produced daily to minimise costs?

**6** The total cost of producing $x$ blankets per day is $(\frac{1}{4}x^2 + 8x + 20)$ dollars, and for this production level each blanket may be sold for $(23 - \frac{1}{2}x)$ dollars.
How many blankets should be produced per day to maximise the total profit?

**9** Consider the manufacture of cylindrical tin cans of 1 L capacity. The cost of the metal used is to be minimised, so the surface area must be as small as possible.

**a** If the radius is $r$ cm, explain why the height $h$ is given by $h = \dfrac{1000}{\pi r^2}$ cm.

**b** Show that the total surface area $A$ is given by $A = 2\pi r^2 + \dfrac{2000}{r}$ cm$^2$.

**c** Use technology to help sketch the graph of $A$ against $r$.

**d** Find $\dfrac{dA}{dr}$. Hence find the value of $r$ which makes $A$ as small as possible.

**e** Sketch the can of smallest surface area.

Assignment:

Exercise 21 D.1 #1, 3, 5, 7