Chapter 5

Sequences and series

- **A** Number sequences
- **B** The general term of a number sequence
- **C** Arithmetic sequences
- **D** Geometric sequences
- **E** Series
- **F** Compound interest
- **G** Depreciation

Syllabus reference: 1.7, 1.8, 1.9

A NUMBER PATTERNS

4 Describe the following number patterns and write down the next three terms:

a 1, 4, 9, 16, ... 25   **b** 1, 8, 27, 64, ...   **c** 2, 6, 12, 20, ...

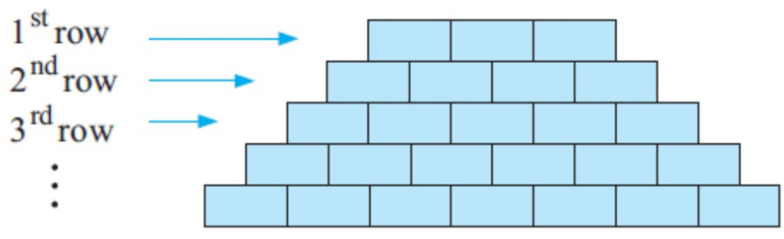
+3 +5 +7

List the first *five* terms of the sequence:

	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$\{2n\}$	2	4	6	8	10
$\{4n-3\}$	1	5	9	13	17
$\{5n+4\}$	9	14	19	24	29
$\{2^n\}$	2	4	8	16	32

B THE GENERAL TERM OF A NUMBER SEQUENCE

Consider the illustrated tower of bricks. The top row, or first row, has three bricks. The second row has four bricks. The third row has five, etc.



If u_n represents the number of bricks in row n (from the top) then

$$u_1 = 3, \quad u_2 = 4, \quad u_3 = 5, \quad u_4 = 6, \quad ......$$

The number pattern: 3, 4, 5, 6, ... is called a **sequence** of numbers.

This sequence can be specified by:

- **Using words**

  The top row has three bricks and each successive row under it has one more brick.

- **Using an explicit formula**

  $u_n = n + 2$ is the **general term** (or **nth term**) formula for $n = 1, 2, 3, 4, 5, ...$ etc.

C ARITHMETIC SEQUENCES

ARITHMETIC SEQUENCES

Number patterns where we **add** (or **subtract**) the same fixed number to get the next number

- Simple interest accumulated amounts at the end of each period.

  For example: on a $1000 investment at 7% simple interest p.a. (per annum) the value of the investment at the end of successive years is: $1000, $1070, $1140, $1210, $1280, ...

- The amount still owed to a friend when repaying a personal loan with fixed weekly repayments.

  For example: if repaying $75 each week to repay a $1000 personal loan the amounts still owing are: $1000, $925, $850, $775, ...

for an **arithmetic sequence** with **first term** u_1 and **common difference** d the **general term** (or nth **term**) is $u_n = u_1 + (n-1)d$.

4 9

Consider the sequence 6, 17, 28, 39, 50, ...

a Show that the sequence is arithmetic.

b Find the formula for its general term.

c Find its 50th term.

d Is 325 a member?

e Is 761 a member?

$$\frac{13}{539}$$

a) 6 17 28 +11 +11 +11

b) $u_n = 6 + (n-1)11$

c) $u_{50} = 6 + (50-1)11$

$u_{50} = 545$

d) $325 = 6 + (n-1)11$

$n =$

A sequence is defined by $u_n = 3n - 2$.

a Show that the sequence is arithmetic. (**Hint:** Find $u_{n+1} - u_n$.)

b Find u_1 and d.

c Find the 57th term.

d What is the least term of the sequence which is greater than 450?

$450 = 3n - 2$

$n = 150.\overline{6}$

n of 151

$u_{151} = 453$

D GEOMETRIC SEQUENCES

GEOMETRIC SEQUENCES

Instead of adding (or subtracting) a fixed number to get the next number in a sequence we sometimes **multiply** (or **divide**) by a fixed number.

Consider investing \$6000 at a fixed rate of 7% p.a. compound interest over a lengthy period. The initial investment of \$6000 is called the principal.

After 1 year, its value is $\$6000 \times 1.07$ {to increase by 7% we multiply to 107%}

After 2 years, its value is $(\$6000 \times 1.07) \times 1.07$
$= \$6000 \times (1.07)^2$

After 3 years, its value is $\$6000 \times (1.07)^3$, etc.

The amounts \$6000, $\$6000 \times 1.07$, $\$6000 \times (1.07)^2$, $\$6000 \times (1.07)^3$, etc. form a geometric sequence where each term is multiplied by 1.07 which is called the **common ratio**.

A few formulae:

for a **geometric sequence** with **first term** u_1 and **common ratio** r, the **general term** (or nth **term**) is $\quad u_n = u_1 r^{n-1}$.

If a, b and c are any consecutive terms of a geometric sequence then

$$\frac{b}{a} = \frac{c}{b} \qquad \{\text{equating common ratios}\}$$

$\therefore \quad b^2 = ac \quad$ and so $\quad b = \pm\sqrt{ac} \quad$ where $\sqrt{ac}$ is the **geometric mean** of a and c.

a Show that the sequence $12, -6, 3, -1.5, ...$ is geometric.

b Find u_n and hence find the 13th term (as a fraction).

$\frac{-6}{12} = -\frac{1}{2}$

$\frac{3}{-6} = -\frac{1}{2}$

$u_n = u_1 r^{n-1}$

$u_n = 12\left(-\frac{1}{2}\right)^{n-1}$

$12\left(\frac{1}{4096}\right)$

$\frac{12}{4096} = \frac{3}{1024}$

Find k given that the following are consecutive terms of a geometric sequence:

a   a b c

a $7,\ k,\ 28$

b $k,\ 3k,\ 20-k$

a b c

$$\frac{3k}{k} = \frac{20-k}{3k}$$

$9k^2 = 20k - k^2$

$10k^2 - 20k = 0$

$10k(k-2)$

c $k,\ k+8,\ 9k$

$$\frac{k+8}{k} = \frac{9k}{k+8}$$

$(k+8)(k+8) = 9k \cdot k$

$k^2 + 16k + 64 = 9k^2$

$0 = 8k^2 - 16k - 64$

$8(k^2 - 2k - 8)$

Find the general term u_n, of the geometric sequence which has:

b $u_3 = 8$ and $u_6 = -1$

c $u_7 = 24$ and $u_{15} = 384$

a Find the first term of the sequence $2, 6, 18, 54, ...$ which exceeds $10\,000$.

Assignment:

Exercises 12	A # 3
	B # 2
	C # 2, 4, 5a-c, 6a, 9
	D # 2, 4, 6, 8a