Complete the following assignment:

Exercises 5

A # 4
B # 6 e-h
C # 2, 4, 5, 8, 9, 11, 14

$41 = u_1 + d(7-1)$

$41 = u_1 + 6d$

$77 = u_1 + d(13-1)$

$77 = u_1 + 12d$

$\frac{36}{6}$

$d = 6$

$u_n = u_1 + d(n-1)$

a) $u_n = 5 + 6(n-1)$

$5 + 6n - 6$

$u_n = 6n - 1$

$41 = u_1 + 6(7-1)$

36

$u_7 = 1 \qquad u_{15} = -39$

$d = \frac{-40}{8} = -5$

$u_n = u_1 + d(n-1)$

$1 = u_1 + -5(7-1)$

$1 = u_1 - 30$

$u_1 = 31$

$u_n = 31 - 5(n-1)$

$31 - 5n + 5$

$u_n = -5n + 36$

a, b, c

$4(-3) - (-3)^2$

$-12 - 9 \quad 3 \quad 3 \quad 3$

$b - a = c - b$

$-12 - 9$

$3k - (4k - k^2) = 3 - 3k$

$-21, -9, 3$

$-k + k^2 = 3 - 3k$

$k = 1 \quad k = -3$

$k^2 + 2k - 3 = 0$

$(k - 1)(k + 3) = 0$

D GEOMETRIC SEQUENCES

GEOMETRIC SEQUENCES

Instead of adding (or subtracting) a fixed number to get the next number in a sequence we sometimes **multiply** (or **divide**) by a fixed number.

Consider investing \$6000 at a fixed rate of 7% p.a. compound interest over a lengthy period. The initial investment of \$6000 is called the principal.

After 1 year, its value is $\$6000 \times 1.07$ {to increase by 7% we multiply to 107%}

After 2 years, its value is $(\$6000 \times 1.07) \times 1.07$
$= \$6000 \times (1.07)^2$

After 3 years, its value is $\$6000 \times (1.07)^3$, etc.

The amounts \$6000, $\$6000 \times 1.07$, $\$6000 \times (1.07)^2$, $\$6000 \times (1.07)^3$, etc. form a geometric sequence where each term is multiplied by 1.07 which is called the **common ratio**.

A few formulae:

for a **geometric sequence** with **first term** u_1 and **common ratio** r, the **general term** (or nth **term**) is $u_n = u_1 r^{n-1}$.

If a, b and c are any consecutive terms of a geometric sequence then

$$\frac{b}{a} = \frac{c}{b} \quad \{\text{equating common ratios}\}$$

$\therefore \quad b^2 = ac$ and so $b = \pm\sqrt{ac}$ where $\sqrt{ac}$ is the **geometric mean** of a and c.

Consider the sequence $8, 4, 2, 1, \frac{1}{2}, \ldots$

a Show that the sequence is geometric.

b Find the general term u_n.

c Hence, find the 12th term as a fraction.

a) $r = \frac{1}{2}$

$u_n = u_1 r^{n-1}$

b) $u_n = 8\left(\frac{1}{2}\right)^{n-1}$

c) $u_{12} = 8\left(\frac{1}{2}\right)^{12-1}$

$u_{12} = \frac{1}{256}$

$k - 1$, $2k$, and $21 - k$ are consecutive terms of a geometric sequence. Find k.

$$\frac{2k}{k-1} = \frac{21-k}{2k}$$

$$4k^2 = (k-1)(21-k)$$

$$4k^2 = 21k - k^2 - 21 + k$$

$$5k^2 - 22k + 21 = 0$$

$$(\quad)(\quad) = 0$$

$k = 3$

$k = 1.4$

Find the first term of the geometric sequence 2, 6, 18, 54, ... which exceeds 10 000.

A geometric sequence has $u_2 = -6$ and $u_5 = 162$. Find its general term.

GEOMETRIC SEQUENCE PROBLEMS

Problems of **growth and decay** involve repeated multiplications by a constant number. We can therefore model the situations using geometric sequences.

The initial population of rabbits on a farm was 50.
The population increased by 7% each week.

a How many rabbits were present after:

  i 15 weeks **ii** 30 weeks?

b How long would it take for the population to reach 500?

Assignment:

Exercises 5 ~~D.1 # 1, 5, 8, 9~~

D.2 # 2, 5 → tomorrow