IB Math Studies 1 BELL WORK

1) Find the first term of the geometric sequence

$2, 6, 18, 54, \ldots$ which exceeds $10\,000$.

2) Find the first term of the geometric sequence

$4, 4\sqrt{3}, 12, 12\sqrt{3}, \ldots$ which exceeds 4800.

Chapter 5

Sequences and series

A Number sequences
B The general term of a number sequence
C Arithmetic sequences
D Geometric sequences
E Series
F Compound interest
G Depreciation

Syllabus reference: 1.7, 1.8, 1.9

E SERIES

A **series** is the addition of the terms of a sequence.

21, 23, 25, 27, ..., 49 is an arithmetic sequence.

$21 + 23 + 25 + 27 + ... + 49$ is an arithmetic series.

For the sequence $\{u_n\}$ the corresponding series is $u_1 + u_2 + u_3 + ...$

The **sum** of a series is the result when we perform the addition.

Given a series which includes the first n terms of a sequence, its sum is $S_n = u_1 + u_2 + u_3 + ... + u_n$.

SUM OF AN ARITHMETIC SERIES

Find the sum of the series:

$$4 + 6 + 8 + \ldots + 26$$

The sum if referred to as S_n.

For this series, list u_1, d, n, and u_n.

$$4 + 6 + 8 + \ldots + 26$$

You try it. Find the sum of the series.

$9 + 12 + 15 + ... + 30$

The sum of an arithmetic series with n terms is

$$S_n = \frac{n}{2}(u_1 + u_n) \qquad \textit{or} \qquad S_n = \frac{n}{2}(2u_1 + (n-1)d).$$

Find the sum of $4 + 7 + 10 + 13 + \ldots$ to 50 terms.

Find the sum of $-6+1+8+15+\ldots.+141.$

GEOMETRIC SERIES

A **geometric series** is the addition of successive terms of a geometric sequence.

If we are adding the first n terms of a geometric sequence, we say we have a **finite geometric series**.

If we are adding all of the terms in a geometric sequence which goes on and on forever, we say we have an **infinite geometric series**.

SUM OF A FINITE GEOMETRIC SERIES

For a finite geometric series with $r \neq 1$,

$$S_n = \frac{u_1(r^n - 1)}{r - 1} \quad \text{or} \quad S_n = \frac{u_1(1 - r^n)}{1 - r}.$$

Find the sum of $2 + 6 + 18 + 54 + \ldots$ to 12 terms.

$$S_{12} = \frac{u_1 (r^n - 1)}{r - 1} \qquad u_1 = 2 \qquad r = 3$$

$$S_{12} = \frac{2(3^{12} - 1)}{3 - 1} = 531{,}440$$

Find a formula for S_n for the first n terms of $9-3+1-\frac{1}{3}+....$

$r = -\frac{1}{3}$

$u_1 = 9$

$n = n$

$$S_n = \frac{9\left(\left(-\frac{1}{3}\right)^n - 1\right)}{-\frac{1}{3} - 1}$$

$$-\frac{3}{4}\left(\frac{9\left(\left(-\frac{1}{3}\right)^n - 1\right)}{-\frac{4}{3}}\right)$$

$$S_n = \frac{-27\left(\left(-\frac{1}{3}\right)^n - 1\right)}{4}$$

A geometric sequence has first term 5 and common ratio 2. The sum of the first n terms of the sequence is 635. Find n.

$$S_n = \frac{u_1(r^n - 1)}{r - 1}$$

$S_n = 635$
$u_1 = 5$
$r = 2$

$$635 = \frac{5(2^n - 1)}{2 - 1}$$

$$127 = 2^n - 1$$

$$128 = 2^n$$

$$\log_2 128 = n$$

$$n = 7$$

Assignment:

5 E.1 # 4, 7

5 E.2 # 2, 3, 6