$\sqrt[3]{x^7}$

$x^{7/3}$

Chapter 8

Logic

Contents:

Syllabus reference: 3.1, 3.2, 3.3, 3.4

D IMPLICATION AND EQUIVALENCE

IMPLICATION

$p \to q$

if p then q

p implies q

If two propositions can be linked with "If .... then ....", then we have an **implication**.

The implicative statement "if p then q" is written $p \Rightarrow q$ and reads "p implies q".

p is called the **antecedent** and q is called the **consequent**.

hypothesis

conclusion

For example: Given p: Kato has a TV set, and q: Kato can watch TV,

we have $p \Rightarrow q$: If Kato has a TV set, then Kato can watch TV.

THE TRUTH TABLE FOR IMPLICATION

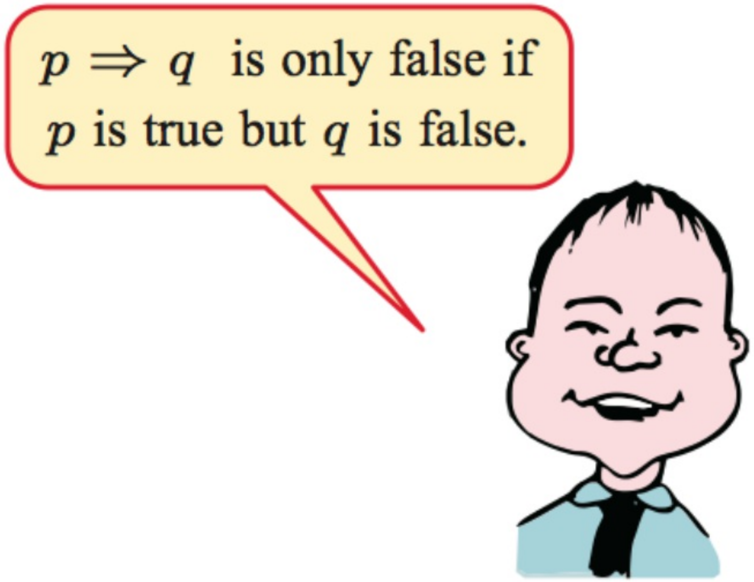
Consider p: It will rain on Saturday, and q: The Falcons will win.

$p \rightarrow q$ If it will rain on Sat, then the Falcons will win

p	q	Scenario	$p \Rightarrow q$
T	T	It rains on Saturday, and the Falcons win. This is consistent with the implicative statement.	T
T	F	It rains on Saturday, but the Falcons do not win. This is inconsistent with the implicative statement.	F
F	T	It does not rain on Saturday, and the Falcons win. This is consistent with the implicative statement, as no claim has been made regarding the outcome if it does not rain.	T
F	F	It does not rain on Saturday, and the Falcons do not win. Again, this is consistent with the implicative statement as no claim has been made regarding the outcome if it does not rain.	T

So, the truth table for $p \Rightarrow q$ is:

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T



EQUIVALENCE

$p \Leftrightarrow q$: $p \to q \wedge q \to p$

If two propositions are linked with ".... if and only if ....", then we have an **equivalence**.

The equivalence "p if and only if q" is written $p \Leftrightarrow q$.

Consider p: I will pass the exam, and q: The exam is easy.

We have $p \Rightarrow q$: *If* I pass the exam, *then* the exam is easy.

$q \Rightarrow p$: *If* the exam is easy, *then* I will pass it.

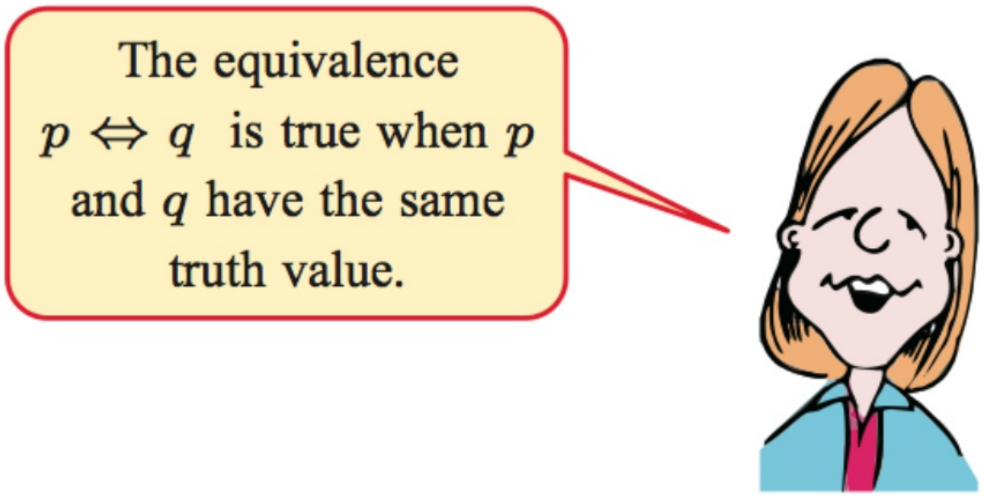
$p \Leftrightarrow q$: I will pass the exam *if and only if* the exam is easy.

$p \Leftrightarrow q$ is logically equivalent to the conjunction of the implications $p \Rightarrow q$ and $q \Rightarrow p$.

THE TRUTH TABLE FOR EQUIVALENCE

$p \Leftrightarrow q$

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Rightarrow q) \wedge (q \Rightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T



Consider the propositions p: It is raining and q: There are puddles forming.
Write the following statements in symbols:

- **a** If it is raining then puddles are forming. $P \to Q$
- **b** If puddles are forming then it is raining. $Q \to P$
- **c** Puddles are not forming. $\neg q$ $\neg q$
- **d** It is not raining. $\neg P$
- **e** If it is not raining, then puddles are not forming. $\neg P \to \neg Q$
- **f** If it is raining, then puddles are not forming. $P \to \neg Q$
- **g** If there are no puddles, then it is raining. $\neg Q \to P$
- **h** It is raining if and only if there are puddles forming. $P \leftrightarrow Q$

$$P \Longleftrightarrow Q$$

$$P \to \neg q$$

Construct truth tables for:

$$p \Rightarrow (p \wedge \neg q)$$

p	q	$\neg q$	$p \wedge \neg q$	$p \to (p \wedge \neg q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

By examining truth tables, show that:

$$\neg p \Rightarrow q \equiv p \vee q$$

Truth table column

Separate column

p	q	$p \vee q$	$\neg p$	$\neg p \to q$

Exercise 8 D: # 2, 5 e,f, 6 a,c