$(p \Leftrightarrow q) \wedge \neg p$

p	q	$p \Leftrightarrow q$	$\neg p$	$(p \Leftrightarrow q) \wedge \neg p$
T	T	T	F	F
T	F	F	F	F
F	T	F	T	F
F	F	T	T	T

$$q \Rightarrow (p \veebar q) = \neg(p \wedge q)$$

p	q	$p \veebar q$	$q \Rightarrow (p \veebar q)$	$p \wedge q$	$\neg(p \wedge q)$
T	T	F	F	T	F
T	F	T	T	F	T
F	T	T	T	F	T
F	F	F	T	F	T

logically equivalent

Chapter 8

Logic

Contents:

Syllabus reference: 3.1, 3.2, 3.3, 3.4

E CONVERSE, INVERSE, AND CONTRAPOSITIVE

Consider the propositions:

p: the triangle is isosceles,

q: two angles of the triangle are equal,

Write the conditional statement $p \rightarrow q$ in words.

If the triangle is isosceles, then two angles of the triangle are equal.

What is the negation of p? Of q?

$\neg p$: the triangle is not isosceles

$\neg q$: two angles of the triangle are not equal

E CONVERSE, INVERSE, AND CONTRAPOSITIVE

If the triangle is isosceles, then two angles of the triangle are equal $p \to q$

p: the triangle is isosceles, q: two angles of the triangle are equal,

	words	symbols
converse:	If two angles of the triangle are equal then the triangle is isosceles	$q \to p$
inverse:	If the triangle is not isosceles, then two angles of the triangle are not equal	$\neg p \to \neg q$
contrapositive:	If two angles of the triangle are not equal, then the triangle is not isosceles	$\neg q \to \neg p$

THE CONVERSE

The converse has truth table:

converse

p	q	$q \Rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	T

THE INVERSE

The inverse has truth table:

p	q	$\neg p$	$\neg q$	$\neg p \Rightarrow \neg q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

This is the same truth table as $q \Rightarrow p$,

so the converse and inverse of an implication are logically equivalent.

THE CONTRAPOSITIVE

The **contrapositive** has truth table:

p	q	$\neg q$	$\neg p$	$\neg q \Rightarrow \neg p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

The truth table for $\neg q \Rightarrow \neg p$ is the same as that for $p \Rightarrow q$,

so the implication and its contrapositive are logically equivalent.

Complete Exercise 8 E