Chapter 9

# Probability

Syllabus reference: 3.5, 3.6, 3.7

## E TREE DIAGRAMS

**Tree diagrams** can be used to illustrate sample spaces

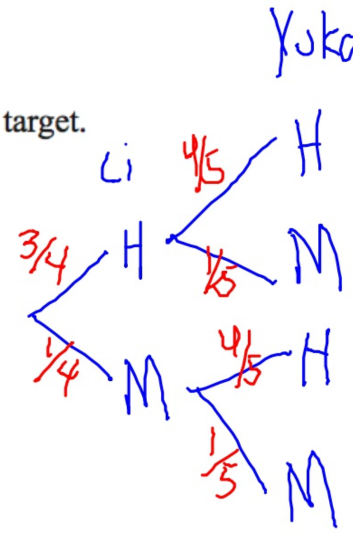
Once the sample space is illustrated, the tree diagram can be used for determining probabilities.

Consider two archers firing simultaneously at a target.

Li has probability $\frac{3}{4}$ of hitting a target and Yuka has probability $\frac{4}{5}$.

The tree diagram for this information is:

H = hit M = miss




$\frac{3}{4} \times \frac{4}{5} = \frac{12}{20}$

$\frac{3}{4} \times \frac{1}{5} = \frac{3}{20}$

$\frac{1}{4} \times \frac{4}{5} = \frac{4}{20}$

$\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$

P(only one Hits target)

$= \frac{3}{20} + \frac{4}{20} = \frac{7}{20}$

Notice from the tree diagram that:

- The probabilities for hitting and missing are marked on the branches.
- There are *four* alternative branches, each showing a particular outcome.
- All outcomes are represented.
- The probability of each outcome is obtained by **multiplying** the probabilities along its branch.

"and" situation: multiply probabilities

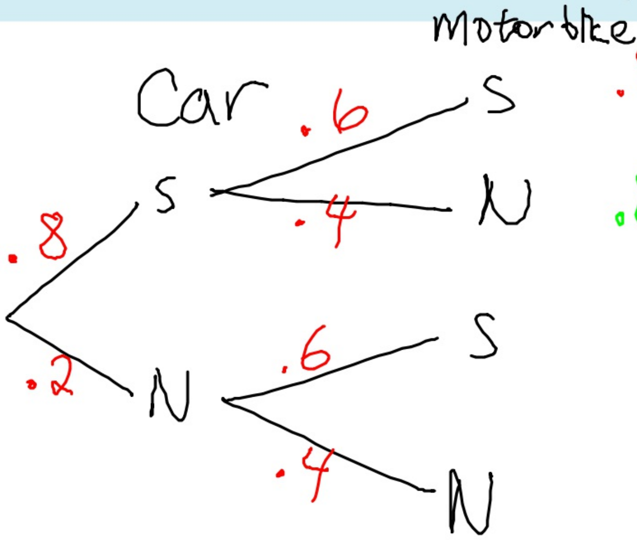
"or" situation: add probabilities

Carl is not having much luck lately. His car will only start 80% of the time and his motorbike will only start 60% of the time.

**a** Draw a tree diagram to illustrate this situation.

**b** Use the tree diagram to determine the chance that:

**i** both will start **ii** Carl can only use his car.

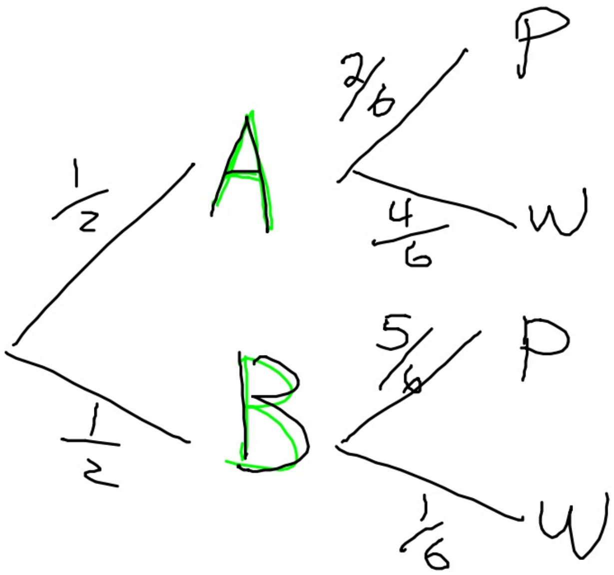



$.8 \times .6 = .48$ (b i)

$.8 \times .4 = .32$ (b ii)

If there is more than one outcome in an event then we need to **add** the probabilities of these outcomes.

Two boxes each contain 6 petunia plants that are not yet flowering. Box A contains 2 plants that will have purple flowers and 4 plants that will have white flowers. Box B contains 5 plants that will have purple flowers and 1 plant that will have white flowers. A box is selected by tossing a coin, and one plant is removed at random from it. Determine the probability that it will have purple flowers.




$$\frac{1}{2} \times \frac{2}{6} = \frac{2}{12}$$

or (+)

$$\frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$$

$$= \frac{7}{12}$$

## F SAMPLING WITH AND WITHOUT REPLACEMENT

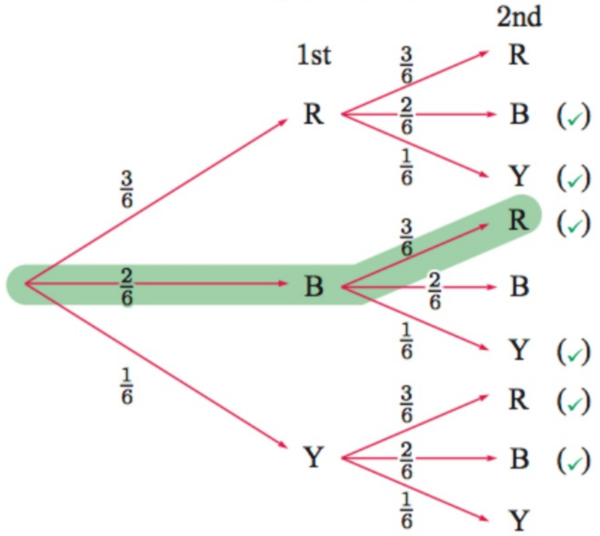
Consider a box containing 3 red, 2 blue, and 1 yellow marble. If we sample two marbles, we can do this either:

- **with replacement** of the first before the second is drawn, or
- **without replacement** of the first before the second is drawn.

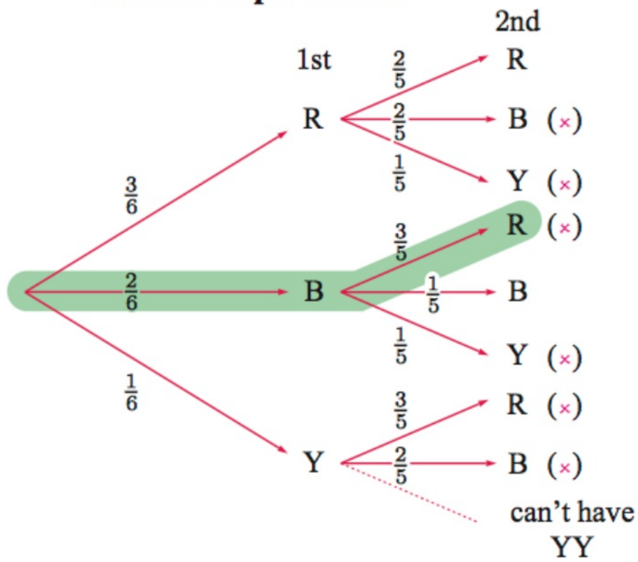
The highlighted branch represents a blue marble with the first draw and a red marble with the second draw. We write this as BR.

Examine how the tree diagrams differ:

**With replacement**




**Without replacement**

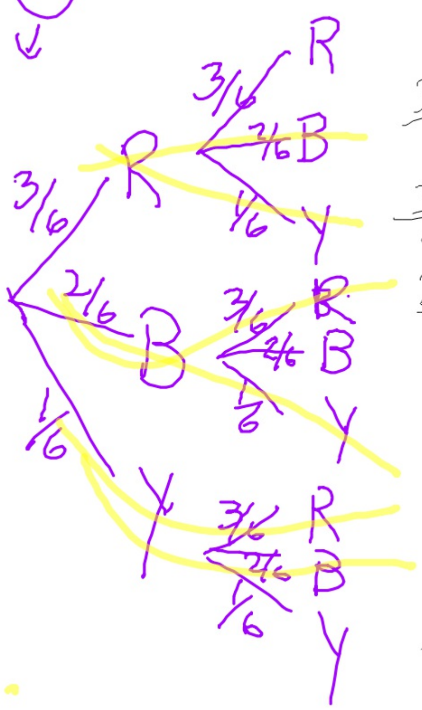



Notice that:

- with replacement
  P(two reds) $= \frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$
- without replacement
  P(two reds) $= \frac{3}{6} \times \frac{2}{5} = \frac{1}{5}$

A box contains 3 red, 2 blue and 1 yellow marble. Find the probability of getting two different colours:

**a** if replacement occurs **b** if replacement does not occur.




$\frac{3}{6} \times \frac{2}{6} = \frac{6}{36}$

$\frac{3}{6} \times \frac{1}{6} = \frac{3}{36}$

$\frac{2}{6} \times \frac{3}{6} = \frac{6}{36}$

$\frac{2}{6} \times \frac{1}{6} = \frac{2}{36}$

$\frac{1}{6} \times \frac{3}{6} = \frac{3}{36}$

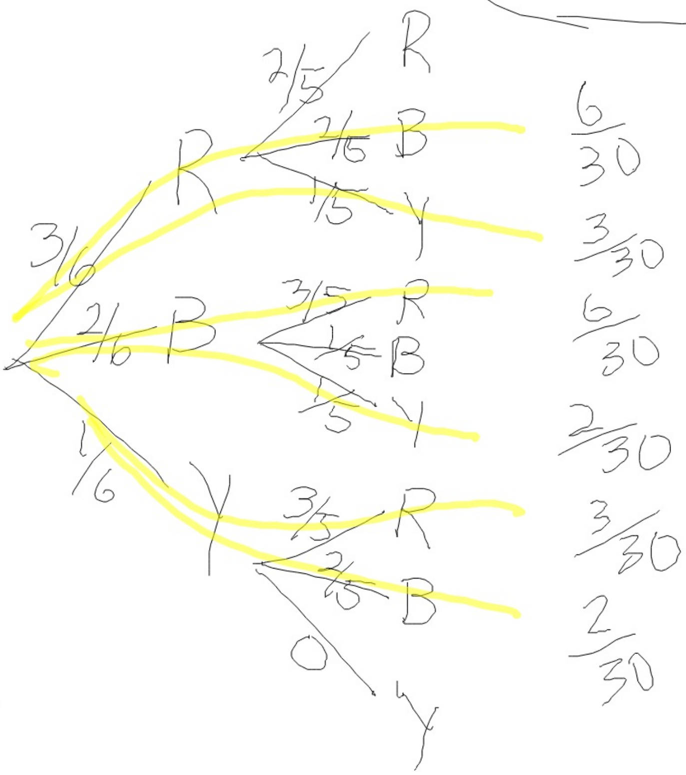
$\frac{1}{6} \times \frac{2}{6} = \frac{2}{36}$

$\frac{22}{36} = \frac{11}{18}$

A box contains 3 red, 2 blue and 1 yellow marble. Find the probability of getting two different colours:

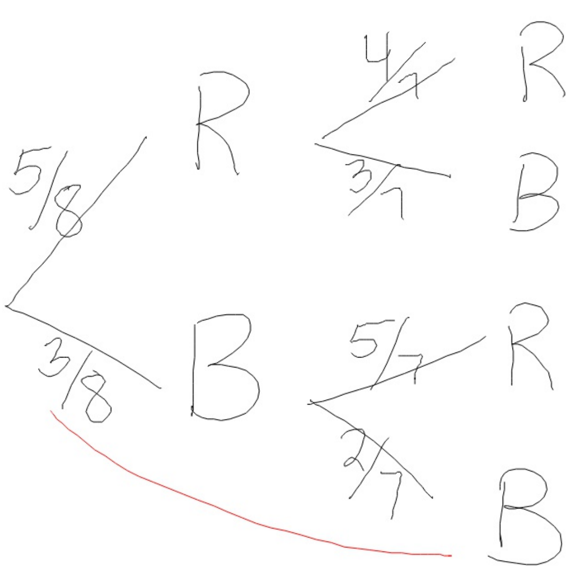
**a** if replacement occurs

**b** if replacement does not occur.




$$= \frac{22}{30} = \frac{11}{15}$$

A bag contains 5 red and 3 blue marbles. Two marbles are drawn simultaneously from the bag. Determine the probability that at least one is red.




$\frac{5}{8}\times\frac{4}{7}=\frac{20}{56}$

$\frac{5}{8}\times\frac{3}{7}=\frac{15}{56}$

$\frac{3}{8}\times\frac{5}{7}=\frac{15}{56}$

$\frac{50}{56}$

$\frac{3}{8}\times\frac{2}{7}=\frac{6}{56}$

$1-\frac{6}{56}=\frac{50}{56}$

$1-P\left(\text{none are red}\right)$

Assignment:

Exercise 9 E # 3, 5, 8<br>F # 1, 2, 7