

Chapter 9

Probability

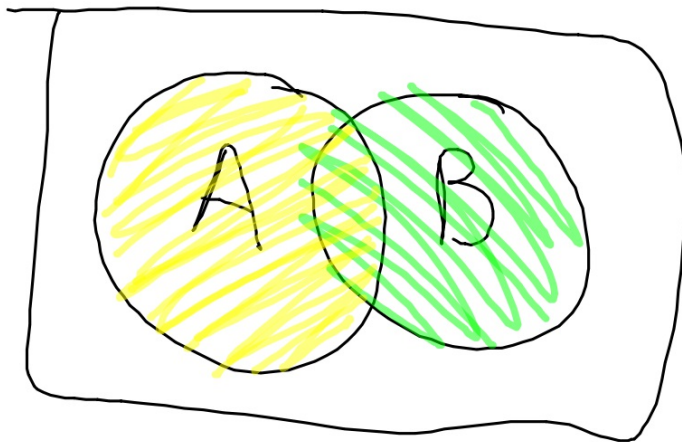
Syllabus reference: 3.5, 3.6, 3.7

- A** Experimental probability
- B** Sample space
- C** Theoretical probability
- D** Compound events
- E** Tree diagrams
- F** Sampling with and without repl
- G** Expectation
- H** Probabilities from Venn diagram
- I** Laws of probability
- J** Conditional probability
- K** Independent events

THE ADDITION LAW

For two events A and B ,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$



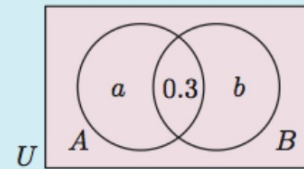
If $P(A) = 0.6$, $P(A \cup B) = 0.7$, and $P(A \cap B) = 0.3$, find $P(B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.7 = 0.6 + P(B) - 0.3$$

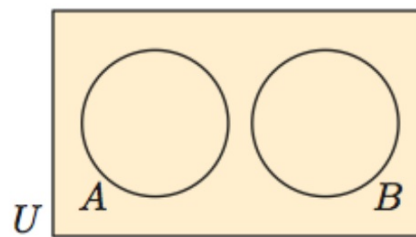
$$0.7 = 0.3 + P(B)$$

$$0.4 = P(B)$$



MUTUALLY EXCLUSIVE OR DISJOINT EVENTS

If A and B are **mutually exclusive** events then $P(A \cap B) = 0$
and so the addition law becomes $P(A \cup B) = P(A) + P(B)$.



Of the 31 people on a bus tour, 7 were born in Scotland (S), and 5 were born in Wales (W).

- a** Are S and W mutually exclusive events?
- b** If a member of the tour is chosen at random, find the probability that he or she was born in:
- i** Scotland
 - ii** Wales
 - iii** Scotland or Wales.

$$\frac{7}{31}$$

$$\frac{5}{31}$$

$$\frac{12}{31}$$

J

CONDITIONAL PROBABILITY

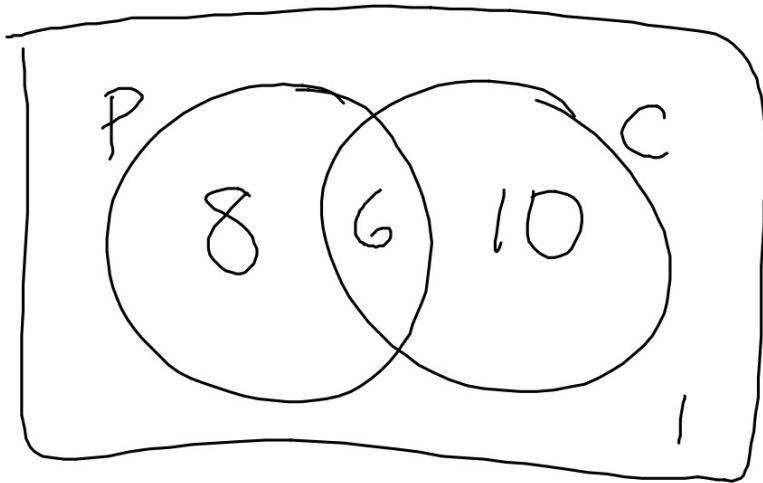
If we have two events A and B , then

$A | B$ is used to represent that “ A occurs knowing that B has occurred”.
 $A | B$ is read as “ A given B ”.

In a class of 25 students, 14 like pizza and 16 like iced coffee. One student likes neither and 6 students like both. One student is randomly selected from the class. What is the probability that the student:

a likes pizza $\frac{14}{25}$

b likes pizza given that he or she likes iced coffee?



$$\frac{6}{16}$$

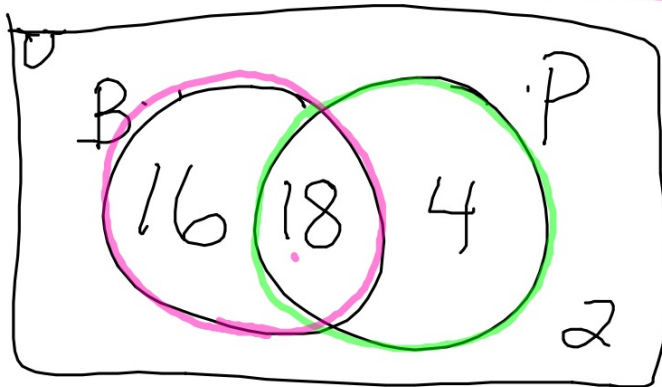
If A and B are events then $P(A | B) = \frac{P(A \cap B)}{P(B)}$.

$$P(B | A) = \frac{P(B \cap A)}{P(A)}$$

$$P(A \cap B) = P(A | B)P(B) \quad \text{or} \quad P(A \cap B) = P(B | A)P(A).$$

In a class of 40 students, 34 like bananas, 22 like pineapple, and 2 dislike both fruits. A student is randomly selected. Find the probability that the student:

- a likes both fruits $\frac{18}{40}$
- b likes at least one fruit $\frac{38}{40}$
- c likes bananas given that he or she likes pineapple $\frac{18}{22}$
- d dislikes pineapple given that he or she likes bananas. $\frac{16}{34}$



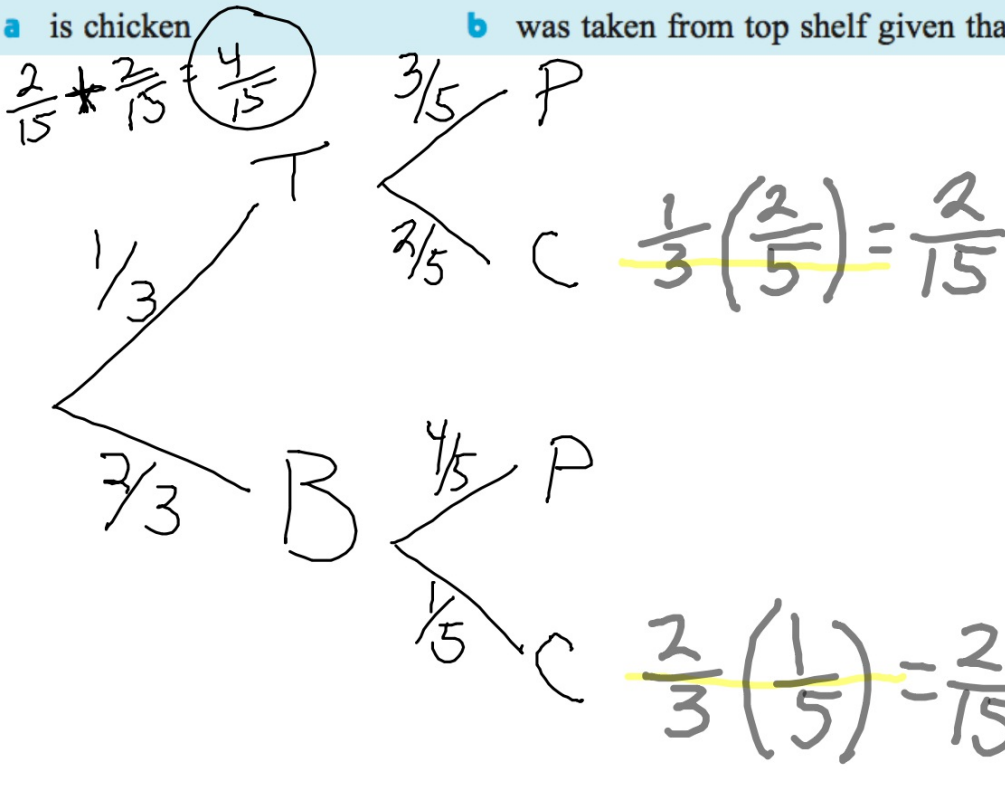
$$\begin{array}{r}
 56 \rightarrow 34 + 22 \\
 - 38 \rightarrow 40 - 2 \\
 \hline
 18
 \end{array}$$

The top shelf in a cupboard contains 3 cans of pumpkin soup and 2 cans of chicken soup. The bottom shelf contains 4 cans of pumpkin soup and 1 can of chicken soup.

Lukas is twice as likely to take a can from the bottom shelf as he is from the top shelf. Suppose Lukas takes one can of soup without looking at the label. Determine the probability that it:

a is chicken

b was taken from top shelf given that it is chicken.



A: top shelf
B: chicken

$P(A|B)$

$$\frac{\frac{2}{15}}{\frac{4}{15}}$$

$$\frac{2}{15} \div \frac{4}{15} = \frac{2}{\cancel{15}} \times \frac{\cancel{15}}{4} = \frac{2}{4} = \frac{1}{2}$$

Exercise

9 I # 1, 2, 5

9 J # 1, 4, 5, 6, 9, 11

